Regression Analysis, Project Template, Residual Plots

## Intuition: Residual Plots

(The attached PDF file has better formatting.)
Updated: February 13, 2006

Jacob: Can you show how to use residual plots to see if the slope parameter $\beta$ is constant?
Rachel: Suppose we have 20 pairs of values for $X$ and $Y$ :

| X | Y | X | Y | X | Y | X | Y |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 18 | 17 | 28 | 22 | 33 | 25 | 33 |
| 10 | 19 | 18 | 29 | 22 | 32 | 25 | 31 |
| 11 | 22 | 20 | 29 | 24 | 31 | 28 | 33 |
| 14 | 23 | 20 | 31 | 25 | 32 | 29 | 33 |
| 15 | 25 | 21 | 31 | 25 | 32 | 30 | 34 |

Ordinary least squares estimation gives

- $\hat{\alpha}=13.671$, with a standard error of 1.427 and a $t$ statistic of 9.58
- $\hat{\beta}=0.74351$, with a standard error of 0.06666 and a $t$ statistic of 11.15

Both coefficients have $p$-values of zero. The $\mathrm{R}^{2}$ is $87.4 \%$ and the adjusted $\mathrm{R}^{2}$ is $86.7 \%$. The regression equation seems fine.

But the residual plots shows the relation is not linear. We use residual plots of the residual vs X and the residual vs the fitted value of Y . In both plots, the residuals look like a carot: $\wedge$.

Jacob: What does that indicate?

Rachel: The slope of the residual line is first positive and then negative. This means that the estimated $\beta$ is too low for low values of $X$ and too high for high values of $X$.

