Jacob: What does it mean that the elasticity varies over a linear curve but is constant over a logarithmic curve?

Rachel: The price elasticity of demand (η) = $\partial Q/\partial P \times P/Q$.

For a linear demand curve, $Q = \alpha - ßP$, so the elasticity (η) = $\partial Q/\partial P \times (P/Q) = -ßP / (\alpha - ßP)$.

~ If P is near zero, the elasticity is close to zero.
~ If Q is near zero, $\alpha \approx ßP$, so the elasticity is close to $-\infty$.

If the relation between two variables is multiplicative, or $Y = \alpha Z^{\beta}$, we take logarithms of both sides to get $ln(Y) = ln(\alpha) + \beta\, ln(Z)$. This is a logarithmic curve.

- β is the derivative of $ln(Y)$ with respect to $ln(Z)$.
- $\partial ln(Y) = \partial Y/Y$ and $\partial ln(Z) = \partial Z/Z$.
- $\partial ln(Y) / \partial ln(Z)$ is the elasticity of Y with respect to Z.

The elasticity is constant over the curve.