

Microeconomics, Module 1, "Consumer's Demand, Producer's Supply, and Equilibrium"

Homework Assignment: Addendum

(The attached PDF file has better formatting.)

Updated: December 21, 2006

{The solution in the posting above is correct. The Jacob - Rachel dialogue below provides the intuition and the method.}

Jacob: How can we grasp the intuition for the maximum tax revenue?

Rachel: Consider the end points. If the tax is \$0 per unit, the total tax revenue is zero.

Jacob: At this tax, the equilibrium price is the intersection of the supply and demand curves

$$200 - 20P = 10P - 10 \Rightarrow 210 = 30P \Rightarrow P = \$7.$$

At $P = \$7$, the quantity demanded is $200 - 20P = 60$.

If the tax is \$3, the total price paid by consumers is $\$7 + \$3 = \$10$.

At a total price of $P = \$7$, the quantity demanded is $200 - 20P = 0$.

How can the tax giving the maximum revenue be \$4.50?

Rachel: Landsburg emphasizes this point. You assume the tax is paid by consumers. Landsburg explains that the incidence of the tax is shared by consumers and producers.

Jacob: Who pays more of the tax?

Rachel: The slope of the demand curve is twice the slope of the supply curve. The curves are linear, so the consumers pay half as much of the tax as producers.

Jacob: Can you show this intuitively?

Rachel: Suppose the tax were \$9 per unit. The demand curve becomes $200 - 20P - 180 = 20 - 20P$. To find P , we equate:

$$20 - 20P = 10P - 10 \Rightarrow 30 = 30P \Rightarrow P = \$1.$$

At $P = \$1$, the *quantity supplied* is $10 \times 1 - 10 = 0$. The *quantity demanded* depends on the total price of $\$1 + \$9 = \$10$. The quantity demanded is zero.

Jacob: Do we infer that at a tax of \$9, the equilibrium quantity is zero, so the total tax revenue is zero?

Rachel: Yes. The supply and demand curves are linear, so the tax revenue is symmetric. The maximum tax revenue occurs at the midpoint of 0 and 9, or \$4.50. The equilibrium quantity at that point is the midpoint of 0 and 60, or $Q = 30$. The tax revenue is $Q \times T = \$135$.

Jacob: How do we solve this by calculus?

Rachel: The demand curve is $Q = 200 - 20P$, and the supply curve is $Q = 10P - 10$. If the tax is T , the demand curve becomes $Q = 200 - 20P - 20T$. The equilibrium quantity is

$$\begin{aligned}200 - 20P - 20T &= 10P - 10 \Rightarrow \\210 &= 30P + 20T \Rightarrow \\30P &= 210 - 20T \Rightarrow P = 7 - \frac{2}{3}T \\Q &= 10P - 10 = 60 - 20T / 3.\end{aligned}$$

The total tax revenue is $T \times Q = 60T - 20/3 T^2$

To maximize the tax revenue, we differentiate with respect to T : $60 - 40/3 T$.

We set this to zero, to get $T = 60 \times 3/40 = 4.50$.

Jacob: What do we learn from this addendum to the homework assignment?

Rachel: Barro discusses this in depth (the macroeconomics on-line course). There is a point of maximum tax revenue. If the government raises the tax rate above this point, it collects less tax, not more tax.

Jacob: Do countries have tax rates above the maximum point?

Rachel: Several European countries are above the maximum point. If the government would lower the tax rate, it would collect more tax.

Jacob: Do we see this in the United States?

Rachel: Clinton raised the tax rate on wealthy people during his first term. The result was that wealthy people paid less tax, because they worked less.

Jacob: If the tax increase caused less tax revenue, why would anyone support it?

Rachel: Barro did an informal survey of his friends. Barro teaches at Harvard, and many of his colleagues favor higher taxes on wealthy persons. Barro asked: "If a tax increase on the wealthy causes the government to collect less total tax, is it good?"

Jacob: I presume they all said no. If the government gets less tax, it has less money to give to the poor or to public programs. How can this be good?

Rachel: On the contrary: many said yes. One goal of high taxes is to redistribute money from wealthy persons to poor persons (or public programs). But there is another purpose. Many people don't like income inequality. The best outcome is to raise the income of the poor so everyone is wealthy. If we can't do this, the second best outcome is to reduce the income of the wealthy.

Jacob: It seems strange that people have this desire.

Rachel: Several European countries have such high tax rates and restrictions on work that no one earns that much after-tax. One might think citizens are upset that their incomes are lower than those in the U.S. or other countries with lower tax rates. But many people are satisfied. They don't want the high income inequality of the U.S.

Jacob: Do people in the U.S. care less about income inequality?

Rachel: The difference in the U.S. is that most people believe they (or their children) can move up by hard work. They may care about income inequality, but they expect to move up and become part of the high earning group.