MA(1), AR(2), AND ARMA(1,1) MODELS

(The attached PDF file has better formatting.)

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The textbook explains the statistical properties of ARIMA processes. This posting gives examples of MA(1), AR(2), and ARMA(1,1) processes in actuarial work, with *intuitive* explanations of when these models are used.

We discuss loss cost trends for stochastic lines of business and short underwriting cycles.

Loss Cost Trends

Autoregressive process: Suppose inflation is highly variable, and we have information about the expected trend. If we observe an 8% trend one year, we forecast an 8% trend the next year as well. This is an autoregressive process with ϕ_1 = 100%.

Jacob: Do we expect ϕ_1 = 100%?

Rachel: Some Western countries target an inflation rate, such as 6% per annum.

- If inflation one year is more than 6%, the central bank may tighten the money supply. We expect inflation the next year to fall toward 6%.
- If inflation one year is less than 6%, the central bank may loosen the money supply. We expect inflation the next year to rise toward 6%.

The time series may be a stationary autoregressive process with $\phi_1 < 100\%$.

Jacob: When might we see a moving average process?

Rachel: Suppose inflation is stable at 10% per annum and we model commercial fire loss cost trends for a small insurer. We expect the trend series to be +10%, +10%, ….

Your student project may take logarithms and first differences of average claim severities.

- The average claim severities may be \$10,000, \$10,000 × 1.10, \$10,000 × 1.10^2, …
- Taking logarithms ⇛ *ln*(\$10,000), *ln*(\$10,000) + *ln*(1.10), *ln*(\$10,000) + 2*ln*(1.10), …
- The first differences are *ln*(1.10), *ln*(1.10), …, ≈ 10%, 10%, …

If losses are not stochastic, the time series is a simple exponential trend. Random loss fluctuations add a moving average part to this time series.

If one year has many large claims (by chance), the observed trend that year may be +12%. The expected trend the next year is +8%, not +10%. This is an MA(1) model with a θ_1 parameter of 100%.

Take heed: The trend is exponential, not linear. The 10% trend is a 10% linear trend after taking logarithms.

Jacob: Do we really expect θ_1 = 100%?

Rachel: Suppose the expected loss cost trend is +10%, and we observe +12% from 20X7 to 20X8. We consider several scenarios:

- The 20X7 average claim severity was the expected value, and the 20X8 average claim severity was 2% higher than expected. We expect the trend from 20X8 to 20X9 to be 2% less than 10%, or 8%.
- The 20X8 average claim severity was the expected value, and the 20X7 average claim severity was 2% lower than expected. We expect the trend from 20X8 to 20X9 to be 10%.

If both scenarios are equally likely, θ_1 may be 50%.

Jacob: These are AR(1) and MA(1) models. When would an ARMA(1,1) model be used?

Rachel: We combine the two stochastic pieces.

- For inflation, we use an AR(1) model with a positive parameter.
- For the stochasticity of fire losses, we add an MA(1) component.

The AR(1) model reflects the stochasticity of inflation. The MA(1) component adds the sampling error of the observed fire losses. The relative size of the components depends on the relative importance of these two forms of stochasticity.

Take heed: The project template on loss cost trends uses AR(1), MA(1), and ARMA(1,1) processes, along with seasonality and structural elements. The optimal trend model is rarely an exponential fit. Many items affect the loss cost trend. Actuary have long sought more accurate methods of projecting trends. The ARIMA processes reviewed in the time series course may improve your trend models.

Underwriting Cycles

Underwriting cycles (and other business cycles) may be modeled by AR(2), ARMA(1,1), ARMA(2,1), and similar models.

Jacob: How do cycles differ from seasonality? They both show an oscillating pattern: higher in some months, quarters, or years, and lower in other months, quarters, or years.

Rachel: They differ in two ways. We compare

- Seasonal toy sales, which have higher fourth quarter than second quarter figures.

- Cyclical auto insurance profits, which may have a four year oscillating pattern.

We assume a four year underwriting cycle to make the comparison clearer.

Seasonality says that fourth quarter sales are higher than average. If second quarter toy sales are low one year, we presume that the true average toy sales are low this year, so fourth quarter sales may also be lower than usual. A cycle says the opposite: if 20X5 insurance profits are low, the cycle may be more severe, and 20X7 profits may be higher than usual.

Insurance industry projections often have a cyclical perspective. After September 2001, many insurers expected higher premiums and strong profits in 2002. Some economists say such projections are not compatible with a competitive market. We do not seek the economic rationale for a cycle, but you may do a student project on insurance industry profits by line of business over the past 60 years to see is a cycle is reasonable.

Seasonality has fixed times; toy sales are high in December and perhaps in November. Cycles are not fixed. A four year cycle may extend into a five year cycle. If a cycle stops one year for exogenous reasons, we expect the cycle to renew with a 1 year lag in the future. We model cycles with ARIMA processes, not with curves of fixed periods.

Illustration: Suppose auto insurance has a four year cycle. If profits are low in 20X5, we expect average profits in 20X6 and high profits in 20X7. If profits are again low in 20X6, we expect average profits in 20X7 and high profits in 20X8.

Jacob: Do we use autoregressive or moving average models for seasonality?

Rachel: We use autoregressive, not moving average models. Autoregressive models deal with expected changes, such as higher than average December sales. Moving average models deal with *unexpected* changes. In the cycle illustration above, we expected average profits in 20X6 but actual profits were low.

Jacob: How do we model a cycle? Suppose an industry has profit cycles, where annual profits relative to long-run mean are 0, +Z, 0, –Z. This looks the same as seasonality; do we use profit(t) = profit(t-4) + ϵ?

Rachel: This has two problems:

- If profit at bottom of cycle was unusually poor, such as –2Z, we expect profit at upturn to be unusually good, such as +2Z. So we use –profit(t-2). This is an autoregressive model.
- If the cycle is delayed one period by an exogenous force, we expect the cycle to continue afterwards with a one period delay. If we have an extra year of zero profits in year 6 of this time series, we expect: 0, +Z, 0, –Z, 0, 0, +Z, 0, –Z.

We might use an ARMA(2,1) process with $\phi_2 = -1$ and $\theta_1 = -1$.

- If the cycle is on schedule, the AR process shows the expected profits.
- If the cycle has a delay, such as the two periods with zero profits, the residual gets the cycle back on track.

Jacob: This illustration seems simplistic. Cycles don't have regular four year periodicity. A cycle may be five, six, or seven years. How do we deal with this uncertainty?

Rachel: We may have two assumptions:

- This year's profits will be like last year's profits, were there no cycle.
- If last year's profits were higher (lower) than those of the year before, we assume the cycle is moving up (down), and this year's profits may be even higher (lower).

We can model this as an AR(2) process. The first assumption is an AR(1) model. If the average profits are 12%, the model may be $y_t = 6\% + \frac{1}{2} y_{t-1} + \epsilon$. The second assumption is an AR(2) process, such as $y_t = \frac{1}{4} (y_{t-1} - y_{t-2}) + \epsilon$. Putting the two together gives

$$y_t = 6\% + \tfrac{3}{4} y_{t-1} - \tfrac{1}{4} y_{t-2} + \epsilon.$$

Jacob: This is still too simple; it doesn't catch turning points of the cycle.

Rachel: More sophisticated ARIMA models may be used for more complex cycles. But the more sophisticated model is not necessarily better. Cycles vary so much that we don't have good models.

Jacob: Can we do a student project analyzing property-casualty underwriting cycles?

Rachel: The data are so haphazard that no ARIMA model fits well and you won't see how to apply the statistical techniques. The project templates for interest rates show how to apply the statistical techniques and arrive at proper inferences.

Take heed: Researchers do not agree on the existence, shape, or periods of underwriting cycles. You can collect insurance industry profit margins by line of business and fit ARIMA processes. You can use simple processes; less you have time series software, you can't fit complex ARIMA processes. Use multiple linear regression to estimate and AR(1) and AR(2) process, and the Yule-Walker equations for MA(1) and ARMA(1,1) processes.