TIME SERIES PROJECT OSCILLATING, MEAN REVERTING, AND CYCLICAL

(The attached PDF file has better formatting.)

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Jacob: What is the difference between oscillating, mean reverting, and cyclical?

Rachel: A non-oscillating mean reverting model moves part way toward the mean. An oscillating process moves toward the mean and over-shoots it.

- An AR(1) model is mean reverting if $-1 < \phi_1 < 1$.
- An AR(1) model is oscillating if $\phi_1 < 0$.

An AR(1) model is stationary if and only if it is mean reverting.

Jacob: Can an AR(1) model be mean reverting, stationary, and oscillatory?

Rachel: Yes. A mean reverting process is stationary. It is also oscillatory if $-1 < \phi_1 < 0$.

Jacob: If a time series has a mean of μ, is oscillatory, and $y_t > \mu$, is $y_{t+1} < \mu$?

Rachel: You should say: "If a time series has a mean of μ, is oscillatory, and $y_t > \mu$, then the *expected value* of $y_{t+1} < \mu$." The time series is stochastic. We forecast the expected values, not the actual values.

Jacob: If a time series has a mean of μ, is mean reverting but not oscillatory, and $y_t > \mu$, is the expected value of y_{t+1} between μ and y_t? Is the probability that $\mu < y_{t+1} < y_t$ more than 50%?

Rachel: The first sentence is correct: If a time series has a mean of μ, is mean reverting but not oscillatory, and $y_t > \mu$, then the expected value of y_{t+1} is between μ and y_t. The second sentence should say: "The probability that $\mu < y_{t+1}$ is more than 50% and the probability that $y_{t+1} < y_t$ is more than 50%."

Jacob: Can we make more exact probabilistic statements? If a time series has a mean of μ, is mean reverting but not oscillatory, and $y_t > \mu$, then the expected value of y_{t+1} is between μ and y_t. Can we make probabilistic statements like: "The probability $y_{t+1} > y_t$ is less than 5%"?

Rachel: We must know the value of y_t and the standard deviation of the time series. If y_t is 1% and σ is 25%, the probability $y_{t+1} > y_t$ is close to 50%.

Jacob: Is cyclical another term for oscillating?

Rachel: Oscillating means the terms alternate about the mean. The autoregressive parameter ϕ_1 is negative. A cyclical pattern, also called a sinusoidal pattern, looks like a sine wave. The autoregressive parameter ϕ_1 is positive, but one or more higher order autoregressive parameters are negative.

Jacob: What is the intuition for an oscillatory process? Do we encounter ARIMA processes with negative sample autocorrelations of lag 1?

Rachel: We may have moving average processes with a negative sample autocorrelations of lag 1. We explain these in other discussion forum postings. They are may occur for loss cost trends in small samples, cycles, and industry wide sales of durable goods.

Take heed: A common cause of negative sample autocorrelations of lag 1 is in the first differences of stationary processes. The common example is

- A time series is a white noise process (or a weak autoregressive process) overlaid on a trend or a long-term cycle.
- The candidate takes first differences to eliminate the trend. This makes the time series stationary, but the first differences have a negative sample autocorrelation of lag 1.
- The ideal solution is to detrend the time series or otherwise offset the long-term cycle. If possible, use a structural model to eliminate the trend.
- If the first differences have negative sample autocorrelations first two or three lags that become zero by the fourth or fifth lag, return to the original time series and detrend the series. You have an easily explained series, that the first differences hide.