THREE DIMENSIONS OF THE LOSS TRIANGLE

(The attached PDF file has better formatting.)

{See also the PDF file on multicollinearity.}

Take heed: The dimensions of the loss triangle are correlated with each other. Review this posting carefully, and be sure you can identify the accident year, development period, and calendar year for each cell of the paid loss triangle. The following principles are important.

- The loss triangle has two dimensions: accident year and development period.
- A third dimension, calendar year, determines the price level (inflation). Calendar year = accident year + development period.
- We use calendar year and development period in the student project. For simplicity, we assume the same exposures in each accident year.
- Calendar year and development period are correlated (ρ = 50%).

The dimensions are hard to grasp at first. They are explained several time in the PDF files on the discussion forum and the illustrative worksheets.

- We use zero-based arrays on the Excel worksheets and in the VBA macros to simplify the formula.
- With one-based arrays, use "year – 1" in the formulas.

Jacob: How do we label the cells of the loss triangle?

Rachel: The method used to label the cells determines the independent variables. We have three axes to label the cells: development period, accident year, and calendar year.

- The development period is the *column*.
- The accident year is the *row*.
- The calendar year is the *diagonal*.

The calendar year = the accident year + the development period.

Take heed: The background posting on paid loss triangles explains the three dimensions.

Take heed: Some loss triangles show calendar year by column and development period by diagonal. The background posting on paid loss triangles discusses this format as well.

Jacob: What is the origin? Is the first accident year, development period, or calendar year 0 or 1?

Rachel: We use an origin of either 0 or 1.

- If we start with origin of 0, the development period and accident year run from 0 to 14. The calendar year equals development period + accident year.
- If we start with origin of 1, the development period and accident year run from 1 to 15. The calendar year equals development period + accident year – 1.

Jacob: Does the origin make a difference?

Rachel: The formulas are slightly easier with an origin of zero, but the intuition might be clearer with an origin of 1. *The illustrative worksheets and postings on the discussion forum use an origin or zero. To follow the illustrative spread-sheet exactly, use an origin of 0.*

Jacob: Do we use all three dimensions for the regression analysis?

Rachel: The three dimensions have perfect multicollinearity. If the regression coefficients are stable, ordinary least squares estimation does not give a unique solution. It gives an infinite number of solutions that are mathematically identical.

We use two dimensions for the regression in the project template on parameter stability:

- The first dimension is the development period.
- The second dimension is the calendar year.

Jacob: Inflation is a calendar year phenomenon. Is the dimension the calendar year or the inflation rate?

Rachel: X_2 is the calendar year index. If the loss triangle covers the 15 years 2000-2014, Year 2000 = 0 and Year 2014 = 14.

β_2 is the annual inflation rate. A loss paid in 2006 has six years of inflation since 2000. We add six years of inflation, or $\beta_2 \times X_2$, to the value of Y.

Jacob: Inflation is multiplicative, not additive. We should multiply by $(1 + \beta_2)$ ^ X_2.

Rachel: The Y values (the dependent variable) is the logarithm of paid losses. The inflation rate in the student project is the continuously compounded inflation rate = *ln*$(1 + \beta_2)$.

Zero-Based Arrays

Jacob: Why do we use zero-based arrays?

Rachel: Definitions:

- X_1 is the development period, and ß$_1$ is the geometric decay.
- X_2 is the calendar year and ß$_2$ is the inflation rate.

- With an origin of zero, the value of Y in the upper left hand cell is α.

- With an origin of 1, the value of Y in the upper left hand cell is $\alpha + ß_1 + ß_2$.

The β's are the same, but the intercept α is different. [*Note:* The background posting on paid loss triangles uses accident year as the first independent variable. The illustrative worksheets and the project template assumes all accident years are the same, except for the effects of inflation.]

Jacob: Do we label the cells (0, 0), (1, 0), (2, 0), …, for the first row?

Rachel: If we use development period by accident year, that would be correct. The first row of paid losses is accident year 0 for each cell and development periods 0, 1, 2, …, 14.

The illustrative worksheet uses development period by calendar year. The first row of cells is (0,0), (1,1), (2, 2), …., (14, 14). The calendar year = accident year + development period.

- Ordinary least squares estimators depend on deviations of the independent variables from their means, so the indexing system is not important.
- Dummy variables and heteroscedasticity use the absolute value of the independent variables, so we are careful about the indexing.

Take heed: Reserving actuaries index the loss triangle by accident year and development period. Using zero-based arrays, the cells are (0, 0), (0, 1), (0, 2), …, (0, 14). The indexing in the student project is different; do not get confused.

Jacob: What is the second row?

Rachel: The second row is (0, 1), (1, 2), (2, 3), …, (13, 14). The last observation in the second row is one development period less than the last observation in the first row.

The last data point in each row has the same calendar year, which is 14 in this scenario.

- The last observation in each row is the current calendar year (= 14 in this scenario).
- Each row has one fewer development period.

Jacob: Do you show the loss triangles on the illustrative spreadsheet?

Rachel: We show illustrative triangles in the posting explaining paid loss triangles. These triangles help you understand the concepts.

- For the student project, begin with a simulation of logarithms of paid losses.
- To forecast reserves, convert the matrix of logarithms to a matrix of paid losses.

Take heed: Your student project may include a section of forecasting future paid losses. This is not required for the student project, since the textbook does not discuss how to forecast expected dollars from ordinary least squares estimates of their logarithms. The procedure seems strange to some candidates. We explain it in a separate posting.