### **P**ROJECT TEMPLATE ON PARAMETER STABILITY

(The attached PDF file has better formatting.)

## OVERVIEW

Classical regression analysis uses regression coefficients that are the same at all points. If the regression coefficient is not the same at all points, we use

- Dummy variables to differentiate discrete periods, such as years 1 5 vs years 6 10.
- Functions of the independent variable for smoothly changing coefficients. For linear changes, we use the square of the independent variable.

The textbook shows several versions of dummy variables. This project template assumes that a slope ( $\beta$ ) coefficient changes, but everything else remains the same.

- ~ The inflation rate may jump from 5% in some years to 25% in other years.
- ~ The inflation rate may change smoothly from 5% to 25%.

The change in the coefficient may be known exogenously or inferred from the data.

- Exogenous knowledge: We may know that inflation is changing.
- Inference: We use residual plots to infer a change in a parameter.

The project template uses residual plots to test if the inflation rate and loss payment pattern are constant.

This student project has several parts:

- Forming regressions (simulating) with ß's that are not constant.
- Analyzing residual plots to determine if the  $\beta$  is constant.
- Re-estimating with dummy variables or squares of independent variables.

The sequence for this project template is:

- Choose the attributes of the scenarios, such as the number of years.
- Run regressions with constant parameters and form residual plots.
- Formulate a scenario, such as inflation increases from 5% to 25%.
- Select a value for  $\sigma$  and simulate data.
  - Use a low or null  $\sigma$  to see the expected results.
  - Use a moderate  $\sigma$  for the student project.
- Form residual plots and interpret the shape of the curve.
- Use dummy variables or functions of the independent variables for a new regression.

Forming Regressions: You simulate data in Excel.

- The course module on error terms explains the normal distribution and its variance.
- Simulate the error term with Excel built-in functions.
- The illustrative worksheets have the needed cell formulas.

*Take heed:* Excel has built-in simulation functions. See the documentation of the illustrative worksheets. Review Modules 3 and 4 of the on-line course, and focus on the distribution of the error term (normal distribution with a mean of zero and a standard deviation of  $\sigma$ ).

*Residual plots:* Plot the residuals against independent variables and interpret the graphs. Use large changes in  $\beta$  values. Focus on three scenarios:

- constant  $\beta$  (straight horizontal line at  $\epsilon = 0$ )
- discrete increase or decrease in  $\beta$  (two line segments: a V or an upside down V)
- smooth increase or decrease in  $\beta$  (convex or concave parabolas)

You can choose a discrete change or a smooth change; you don't have to do both.

- If you have mastered the regression material, the work is straight-forward.
- If you are hazy about regression analysis, the project helps you grasp the material.

*Dummy variables:* Write the regression equation using an additional variable. Re-estimate the ordinary least squares estimators and the  $R^2$ .

- For a discrete change, use dummy variables.
- For a continuous (linear) change, use the square of an explanatory variable.

Take heed: The textbook shows several ways to use dummy variables.

- This project template uses a change in the slope parameter, not the intercept.
- The project uses cumulative inflation and loss payment decay, causing a change in the intercept as well.

Test whether the dummy variable or third independent variable is significant:

- The *t* statistic for the additional variable should be significant.
- The *F* statistic for the restricted vs unrestricted equations should be significant.

If the dummy variable fully explains the change in the slope coefficient, the residual plot should be a flat horizontal line at  $\varepsilon$  = 0.

You will find it helpful to discuss these topics on the discussion forum. A bit of discussion may help you get past a problem in the student project.

# INFLATION RATES AND LOSS PAYMENT PATTERNS

*Jacob:* The project template examines changes in the inflation rate. What about changes in the loss payment pattern?

*Rachel:* This project template examines the constancy of the ß's for the inflation rate or the loss payment pattern. You choose which  $\beta$  to examine. The method is the same for both.

### **G**RAPHICS AND **S**TUDENT **P**ROJECTS

*Jacob:* The statistics textbook is mostly algebra. Why do the project templates focus so much on graphics?

*Rachel:* The project templates emphasize items that are hard to explain in a textbook, such as interpreting residual plots (regression analysis) or correlograms (time series). The SOA stresses that statistics is not just book knowledge. Homework assignments and final exams are not enough to develop skills for interpreting statistical graphs.

#### SIGNIFICANCE AND PARAMETER CHANGES

*Jacob:* Regression analysis has various tests of significance: t values and Z values and *p* values. Aren't those more exact than the residual plots?

Rachel: The tests of significance do not reveal if a regression coefficient is constant.

*Explanation:* Suppose the ordinary least squares estimator for the inflation parameter is +15%, with a *p*-value of 2%, and the regression equation has an  $R^2$  of 80%. You must decide whether the true inflation parameter is 15% each year or changes over the years.

- A *t* test doesn't help much. If the true inflation rate changes from 10% to 20% over the years, the *p*-value may increase to only 3%.
- The R<sup>2</sup> relates to overall variance, not changes in the inflation rate. If the true inflation rate changes from 10% to 20%, the R<sup>2</sup> may change to 70%.