TS Module 17: Forecasting bounds HW

(The attached PDF file has better formatting.)

Homework assignment: Random walk with drift

An insurer's capital follows a random walk with a drift of $10 million a month and a volatility of $40 million a month. The initial capital is $200 million.

A random walk is an ARIMA(0,1,0) process. The capital *changes* are a white noise process, with a mean μ of $10 million a month and a σ of $40 million a month.

A. What is the distribution of capital after one month? (What is the type of distribution, such as normal, lognormal, uniform, or something else? Use the characteristics of a white noise process. What is the mean of the distribution after one month? Use the starting capital and the drift. What is the standard deviation after one month? The volatility is the standard deviation per unit of time, not the variance per unit of time. It is σ, not σ^2.)
B. What is the distribution of capital after six months? (The serial correlation is zero, so the capital changes are independent and the variances are additive. Derive σ^2 for one month from σ, add the σ^2's for six months, and derive the σ after six months.)
C. What is the distribution of capital after one year?
D. What are the probabilities of insolvency at the end of six months and one year? (You have a distribution with a mean μ and a standard deviation σ. Find the probability that a random draw from this distribution is less than zero. Use the cumulative distribution function of a standard normal distribution. Excel has a built-in function for this value.)
E. At what time in the future is the probability of insolvency greatest? (Write an equation for the probability as a function of (i) the mean of the distribution at time *t* and (ii) the standard deviation of the distribution at time *t*. To maximize this probability, set its first derivative to zero, and solve for *t*.)