

## TS Module 18: Forecast updates and weights HW

(The attached PDF file has better formatting.)

*Homework assignment: ARIMA(0,1,1) forecasts*

An ARIMA(0,1,1) model for a time series of 100 observations,  $y_t$ ,  $t = 1, 2, \dots, 100$ , has  $\theta_1 = 0.4$ .

- The forecast of the next observation,  $y_{101}$ , is 25.
- The actual value of  $y_{101}$  is 26.
- The forecast of the next observation,  $y_{102}$ , is 26.
- The actual value of  $y_{102}$  is 26.

We continue to use the same ARIMA model. That is, we don't re-estimate the parameters with the additional data. We forecast  $y_{103}$ , the ARIMA value in the next period.

- A. From the actual and forecasted values of  $y_{101}$ , derive the residual for the ARMA model of the first differences.
- B. From the actual value of  $y_{101}$  and the forecasted value of  $y_{102}$ , derive the forecasted value for Period 102 for the ARMA model of the first differences.
- C. This forecasted value for Period 102 is a function of  $\mu$ ,  $\theta_1$ , and the residual for Period 101. Derive the  $\mu$  (mean) of the ARMA model of first differences.
- D. From the actual and forecasted values of  $y_{102}$ , derive the residual for the ARMA model of the first differences for Period 102.
- E. Using this residual, determine the forecasted first difference for the next period.
- F. From the forecasted first difference, derive the forecasted value of the original time series.

The values of  $\mu$  and  $\theta_1$  are the coefficients of the ARMA process for the first differences. (Cryer and Chan use  $\theta$  for an MA(1) process, not  $\theta_1$ .)