TS Module 19: Seasonal models basics HW

(The attached PDF file has better formatting.)

## Homework assignment: auto insurance seasonality

This homework assignment applies ARIMA modeling to auto insurance policy counts. The same logic applies to other lines of business as well. The homework assignment has three parts, each adding a piece to the insurance scenario.

Part A: Seasonality

The population in State W, the number of drivers, and the number of cars are stable.

- An auto insurer sells twelve month policies in State W.
  - Its renewal rate is 90% on average.
  - The actual renewal rate varies with its prices and those of its competitors.

For Part A of the homework assignment, all insurers charge the same state-made rates.

- Random fluctuation determines how much new business each insurer writes.
- You model policies written each month (both new and renewal) with an ARIMA process.
- A. How would you model renewal policies? Do you use a  $\phi$  (autoregressive) parameter or a  $\theta$  (moving average) parameter?
- B. Is the process stationary?
- C. If the renewal rate were 100%, would the process be stationary?

*Intuition:* If the insurer writes more policies in January 20X1, it writes more policies in January 20X2, January 20X3, and so forth. A random fluctuation dies our slowly. What is the autocorrelation function? Is this a stationary time series?

Part B: We add a free market to this exercise.

- The insurer competes in a free market.
- The insurer revises its rates each year, and its competitors revise their rates at other dates during the year.

New policies sold (new business production) depends on the insurer's relative rate level compared with its competitors. If its rates are lower (higher) than its competitors', it writes more (fewer) new policies.

- The insurer does not expect higher or lower rates than its competitors charge.
- At any time, its rates may be higher or lower than its competitors charge, so its market share may grow or shrink.

- D. How would you change the model? Do you add an autoregressive or a moving average term? Note that rate changes occur once a year, so if the insurer has high (low) rates now, it will probably have high (low) rates next month.
- E. Does the free market increase or decrease the variance of the process? (When firms charge different prices and revise their prices periodically, is market share more or less variable?)

Part C: The insurer revises rates if its policy count is higher or lower than expected.

- If the insurer writes more policies than expected, it is afraid that its rates are too low, and it files for a rate increase.
- If the insurer writes fewer policies than expected, it is afraid that its rates are too high, and it files for a rate decrease.
- F. If this rate change occurs immediately, how should you change the model? Do you add an autoregressive or a moving average term? (In practice, rate filings take several months to be approved. Assume this rate change takes effect immediately.)
- G. If this rate change has a one month lag, how should you change the model? (For this part of the homework assignment, assume the insurer compares its actual vs expected policy count at the end of each month and changes the rate beginning either the next day or one month later. Use whichever assumption you prefer.)