TS Module 15 Mean reversion intuition

(The attached PDF file has better formatting.)

- The time series courses emphasizes the intuition of ARIMA processes.
- Exam problems use scenarios and probabilities to test the intuition.

Autoregressive processes have mean reversion.

Question 1.1: Mean reversion

An AR(1) model of 90 day Treasury bill yields has a mean of 5.00%, φ_1 of 0.60, and σ = 0.4% per month. The values are for the first day of each month.

The date now is January 15, 20X8. Let

- A_T = the *absolute value* of the difference between January's Treasury bill yield and 5%.
- A_{T+1} = *absolute value* of the difference between February's Treasury bill yield and 5%.

For which of the following values of the January 20X8 90 day Treasury bill yield is the probability *greatest* that $A_{T+1} > A_T$?

A. 4.3%
B. 4.6%
C. 4.9%
D. 5.2%
E. 5.5%

Answer 1.1: C

If the problem asks: "For which value of the January 20X8 90 day Treasury bill yield is the probability *greatest* that $y_{T+1} > y_T$," we choose the *lowest* value of y_T (Choice A).

For the *absolute* values, use the following reasoning. If $y_t = \mu$, the difference now is zero. The time series is stochastic with continuous values, so the probability that the value will again be the mean next period is infinitesimally small (i.e., close to zero). If the time series has discrete values, the probability is low. The absolute value of the difference from the mean increases, whether the actual interest rate increases or decreases..

The mean reversion is proportional, so the farther y_t now is from the mean, the more it moves toward the mean. The stochastic term may still move it away from the mean. But the stochastic term is constant for all values of the time series (since it is stationary). As the deterministic movement toward the mean increases, the less likely it will be offset by the stochastic movement.
Stationary autoregressive processes show mean reversion.

If the current value is not the mean, the forecasts eventually move toward the mean. Suppose the mean is μ and the most recent value of the time series is y_T.

- For an AR(1) model, the mean reversion is immediate. Next period's forecast is between μ and y_T.
- For an AR(p) model, the mean reversion may be delayed p-1 periods. The p periods ahead forecast is between μ and y_T.

Question 1.2: Mean reversion

An AR(1) model of 90 day Treasury bill yields has a mean of 5.00%, φ_1 of 0.60, and σ = 0.4% per month. The values are for the first day of each month.

The date now is January 15, 20X8. Let

- A_T = the absolute value of the difference between January's Treasury bill yield and 5%.
- A_{T+1} = absolute value of the difference between February's Treasury bill yield and 5%.

For which of the following values of the January 20X8 90 day Treasury bill yield is the probability *smallest* that $A_{T+1} > A_T$?

A. 4.3%
B. 4.6%
C. 4.9%
D. 5.2%
E. 5.5%

Answer 1.2: A

To grasp the intuition, reason through the following sequence.

If the January 20X8 Treasury bill yield is 5%, A_T is zero. In this exercise, the Treasury bill yield is an ARMA process with a error term. The probability that the February 20X8 Treasury bill yield is also 5% is zero (with no rounding) and very small even with rounding. It is likely that $A_{T+1} > A_T$.

If the January 20X8 Treasury bill yield is 5.01%, A_T is 0.01%. The forecast for February is 5.006%. The actual February yield has a normal distribution with a mean of 5.006% and a standard deviation of 0.4%. The 95% confidence interval for the February 20X8 Treasury bill yield is 4.206% to 5.806%. The probability that the February yield is between 4.99% and 5.01% is very small. We can compute the exact probability with a cumulative density function for the normal distribution. The exam problems do not ask for precise probabilities.

If the January 20X8 Treasury bill yield is 10%, A_T is 5%. The forecast for February is 8%. The actual February yield has a normal distribution with a mean of 8% and a standard deviation of 0.4%. The 95% confidence interval for the February 20X8 Treasury bill yield is 7.2% to 8.8%. The probability that the February yield is between 0% and 10% is almost 100%. This probability is the cdf for ±5 standard deviations from the mean.