

TS Module 6 Stationary autoregressive processes

(The attached PDF file has better formatting.)

Time series practice problems autoregressive process

Understand the difference between moving average and autoregressive processes.

Insurance renewal rates are an autoregressive process. For a renewal rate of R and expected new business of K policies each year, the number of policies forms an AR(1) process with $\phi = R$ and $\theta_0 = K$: $Y_{t+1} = Y_t \times R + K + \epsilon$

Sales of durable products often form a moving average process. Suppose expected sales of new cars is 10,000 each year. If 10,000 + Z cars are sold in 20X1, demand is lower in 20X2, and only 10,000 – $\frac{1}{2} Z$ are expected to be sold.

*Question 1.2: Auto Insurance Policy Volume

An insurer uses an ARIMA process to model its auto insurance policy volume. Each year, 8% of policyholders lapse for non-payment of premium, 1.5% are canceled, 3.5% stop driving, and the rest (85%) renew their policies.

- The insurer expects the same number of new policyholders each year.
- The mean number of policyholders (*both new and renewal*) is 100,000.

If the actual policies issued in the previous year was higher or lower than expected, we expect a higher or lower number of policies issued in the current year that is half as large.

Illustration: Suppose the forecasts for new policies in 20X1 and 20X2 are 10,000 each. If the actual new policies written in 20X1 are 10,000 + Z , the expected new policies written in 20X2 are 10,000 + $\frac{1}{2} Z$.

Which of the following is a reasonable time series for this scenario?

- A. $y_t = 15,000 + \epsilon_t + 0.5\epsilon_{t-1} + 0.85y_{t-1}$
- B. $y_t = 100,000 + 0.5\epsilon_t + 0.5\epsilon_{t-1} + 0.25y_{t-1}$
- C. $y_t = 15,000 + \epsilon_t - 0.5\epsilon_{t-1} + 0.85y_{t-1}$
- D. $y_t = 100,000 + 0.5\epsilon_t - 0.5\epsilon_{t-1} + 0.25y_{t-1}$
- E. $y_t = 15,000\epsilon_t - 0.5\epsilon_{t-1} + 0.85y_{t-1}$

Answer 1.2: A

The insurer expects its sales programs to have a temporary effect. If the number of cars in the state is 1 million and 10 insurers of equal size compete, a rate reduction one year increases the number of cars insured that year. If insurers file for rate changes once a year, it takes time for other insurers to match its rate reduction. In half a year, about half the other insurers have matched its rate reduction, so we assume a moving average parameter of -0.5 (a moving average coefficient of $+0.5$).

The autoregressive parameter is the renewal rate of 85%. The value of δ (θ_0) is the mean of 100,000 times the complement of the renewal rate.

Jacob: What is the insurance difference between a moving average process with θ_1 of -0.5 and an autoregressive process with φ_1 of $+0.5$?

Rachel: The moving average process has a temporary effect. The effect lasts one period and disappears. The autoregressive process has a permanent effect. It is strongest the first period and it dissipates in later periods, but it continues to affect sales.