TS Module 7 Stationary mixed processes

(The attached PDF file has better formatting.)

Time series practice problems residuals

*Question 7.1: Expected Value

An ARMA(2,4) process has moving average parameters $\theta_j = 0.5^j$ for j = 1 to 4 and autoregressive parameters $\phi_k = 0.6^k$ for k = 1 to 2. The parameter $\delta = \theta_0 = 0$, and $\sigma^2 = 1$. All values for t < 12 were the mean, and all residuals for t < 12 were zero.

If $\epsilon_{12} = 1$, what is the expected value of $(\epsilon_{13} - \epsilon_{14}) \times (\epsilon_{13} + \epsilon_{14})$?

A. –0.5
B. –0.25
C. 0
D. +0.11
E. +0.42

Answer 7.1: C

$(\epsilon_{13} - \epsilon_{14}) \times (\epsilon_{13} + \epsilon_{14}) = \epsilon_{13}^2 - \epsilon_{14}^2$

The expected value of the squared residual is constant.

*Question 7.2: Error Terms

A correctly specified ARMA(p,q) process has moving average parameters $\theta_j = 0.5^j$ for j = 1 to 4 and autoregressive parameters $\phi_k = 0.366^k$ for k = 1 to 2. The parameter $\delta = \theta_2 = 5$, and $\sigma^2 = 1$.

Let ϵ_j be the residual for the j^{th} value. What is the expected value of $\rho(\epsilon_1,\epsilon_2)$?

A. –1/6
B. –1/12
C. 0
D. 1/12
E. 1/6

Answer 7.2: C

If the time series process is *correctly specified*, any serial correlation is eliminated by the moving average parameters.

The expected value of the correlation of the residuals depends on the serial correlation of the time series. If the time series is correctly specified, this value is zero. In practice, many time series do not eliminate all the serial correlation; many items can cause serial correlation.

Illustration: Sales figures are affected by inflation, population growth, changes in income, changes in taste, introduction of new products, and a host of other factors. Most of these are serially correlated, and they cause serial correlation in the time series of sales figures. We can not always eliminate the serial correlation with a moving average or autoregressive model.

*Question 7.3: Error Terms

ϵ_j is the residual for the j^{th} value in an AR(1) process with $\phi = 0.5$. If the expected *variance* of ϵ_j is 2 and the actual *value* of ϵ_j is 4, what is the expected *value* of ϵ_{j+1}?

A. 0
B. 1
C. 2
D. 3
E. 4

Answer 7.3: A

The expected value of the residual is zero, regardless of the past values, as long as the process is correctly specified.