

Module 12: Statistical inference for multiple regression

(The attached PDF file has better formatting.)

Multiple regression practice problems

*Question 12.1: Variance of beta

A multiple regression model is $y_j = \alpha + \beta_1 x_{j1} + \beta_2 x_{j2} + \dots + \beta_k x_{jk} + \varepsilon_j$

- σ_ε is the standard error of the regression.
- S_j is the variance of explanatory variable X_j .
- R_j^2 is the R^2 for explanatory variable j regressed on the other explanatory variables.

Which is the correct expression for the variance of the estimator for β_j ?

- A. $V(B_j) = \frac{1}{1-R_j^2} \times \frac{\sigma_\varepsilon^2}{S_j^2}$
- B. $V(B_j) = \frac{1}{1-R_j^2} \times \frac{\sigma_\varepsilon^2}{(n-2)S_j^2}$
- C. $V(B_j) = \frac{1}{1-R_j^2} \times \frac{\sigma_\varepsilon^2}{(n-1)S_j^2}$
- D. $V(B_j) = \frac{1}{R_j^2-1} \times \frac{\sigma_\varepsilon^2}{(n-1)S_j^2}$
- E. $V(B_j) = \frac{R_j^2}{1-R_j^2} \times \frac{\sigma_\varepsilon^2}{(n-1)S_j^2}$

Answer 12.1: C

Know the formulas for the variance and standard deviation of the least squares estimators of the regression coefficients. Focus on the meaning of each variable and the effects, such as “What does R_j mean? If R_j increases, does the variance of B_j increase or decrease?”

*Question 12.2: F-Test

- RegSS is the regression sum of squares in Fox's text (other authors use RSS)
- RSS is the residual (error) sum of squares in Fox's text (other authors use ESS)
- TSS is the total sum of squares
- n is the number of data points in the sample
- k is the number of explanatory variables (not including the intercept)

An F-statistic testing the hypothesis that all the slopes (β 's) are zero has the expression

- A. $\frac{\text{RegSS} / k}{\text{RSS} / (n - k - 1)}$
- B. $\frac{\text{RegSS} / (n - k - 1)}{\text{RSS} / k}$
- C. $\frac{\text{RSS} / (n - k - 1)}{\text{RegSS} / k}$
- D. $\frac{\text{RegSS} / k}{\text{RSS} / (n - k - 2)}$
- E. $\frac{\text{RSS} / k}{\text{RegSS} / (n - k - 1)}$

Answer 12.2: A

Take heed: The formula for the F statistic can be written using RSS, RegSS, or R^2 . The three formulas are equivalent. Know all three for the final exam.

*Question 12.3: Degrees of freedom of F-statistic

A regression model has 14 data points, 3 explanatory variables (β 's), and an intercept.

An F-test for the null hypothesis that **2** slopes are 0 has how many degrees of freedom?

- A. 3 and 10
- B. 2 and 10
- C. 4 and 11
- D. 3 and 11
- E. 2 and 11

Answer 12.3: B

Degrees of freedom = q and $(n - k - 1)$ (p119)

*Question 12.4: Bias

A statistician regresses Y on two explanatory variables X_1 and X_2 but does not use a third explanatory variable X_3 . Under which of the following conditions will β_2 be biased?

- A. $\rho(Y, X_3) = 0$ and $\rho(X_2, X_3) \neq 0$
- B. $\rho(Y, X_3) \neq 0$ and $\rho(X_2, X_3) = 0$
- C. $\rho(Y, X_3) \neq 0$ and $\rho(X_2, X_3) \neq 0$
- D. $\rho(Y, X_2) \neq 0$ and $\rho(X_2, X_3) \neq 0$
- E. $\rho(Y, X_2) = 0$ and $\rho(X_2, X_3) \neq 0$

Answer 12.4: C