TS Module 12: Parameter estimation Yule-Walker equations

(The attached PDF file has better formatting.)

Use the Yule-Walker equations to derive initial estimates of the ARMA coefficients. Know how to solve the Yule-Walker equations for AR(1), AR(2), and MA(1) processes.

- A student project might also use Yule-Walker equations for MA(2) and ARMA models.
- For the final exam, focus on the equations for AR(1), AR(2), and MA(1) models.

Exercise 1.1: MA(1) model and Yule-Walker equations

An MA(1) model has an estimated ρ_1 of –0.35. What is the Yule-Walker initial estimate for θ_1 if it lies between –1 and +1?

Solution 1.1: An MA(1) model has $\rho_1 = \dfrac{-\theta_1}{\left(1+\theta_1^2\right)}$.

We invert the equation to get $\theta_1 = \dfrac{-1 \pm \sqrt{1-4\rho_1^2}}{2\rho_1}$

We compute (–1 + (1 – 4 × $035^2)^{0.5}$) / (2 × –0.35) = 0.408

The final exam uses multiple choice questions. To avoid arithmetic errors, after solving the problem, check that it gives the correct autocorrelation.

The table below shows selected MA(1) values for ρ_1 and θ_1. Note several items:

For a given value of ρ_1, two values of θ_1 may solve the Yule-Walker equation. The exam problem may specify bounds for θ_1, such as an absolute value less than one. The textbook expresses this as the MA(1) model is invertible.

For an invertible MA(1) model, ρ_1 and θ_1 have opposite signs, reflecting the sign convention for the moving average parameter.

Know several limiting cases.

- As $\theta_1 \rightarrow$ zero, $\rho_1 \rightarrow$ zero
- As $\theta_1 \rightarrow$ one, $\rho_1 \rightarrow$ negative one half (–0.5)
- As $\theta_1 \rightarrow$ infinity, $\rho_1 \rightarrow$ zero

θ_1	ρ_1	θ_1	ρ_1
0.1	-0.0990	-0.1000	0.0990
0.2	-0.1923	-0.2000	0.1923
0.3	-0.2752	-0.3000	0.2752
0.4	-0.3448	-0.4000	0.3448
0.5	-0.4000	-0.5000	0.4000
0.6	-0.4412	-0.6000	0.4412
0.7	-0.4698	-0.7000	0.4698
0.8	-0.4878	-0.8000	0.4878
0.9	-0.4972	-0.9000	0.4972