TS Module 14 Model diagnostics

(The attached PDF file has better formatting.)

Time series practice problems model diagnostics

*Question 14.1: Selecting the model

The sample autocorrelations for lags of 1, 2, 3, 4, and 5 from a time series of 900 observations are

25%, 24%, 24.5%, 23.5%, 22.5%

Which of the following is the most likely model (of the following five) for the time series?

A. non-stationary
B. white noise
C. stationary moving average of order 1
D. stationary autoregressive of order 1
E. stationary ARMA(1,1)

Answer 14.1: A

Statement B: If this were a white noise process, the sample autocorrelations would be close to zero, with a standard deviation of 1/30 = 3.3%. The observed autocorrelations are too high.

Statement C: A stationary moving average process of order 1 has white noise autocorrelations for lags 2 and higher. They would be close to zero with a standard deviation of 3.3%.

Statement D: A stationary autoregressive process of order 1 has geometrically declining autocorrelations for lags 2 and higher; there is no decline in the observed autocorrelations.

Statement E: An ARMA(1,1) model is similar to an AR(1) model for the autocorrelations.

Statement A: For a non-stationary process, the autocorrelations may stay high.

*Question 14.2: Diagnostics

An actuary examines a the stock price of ABC Corporation for the 252 trading days in 20X7. How might the actuary test the hypothesis that the stock prices form a process $y_t = 1.01 \times y_{t-1} \times \sigma$, where σ is a lognormally distributed random variable?

A. Assume the time series is a white noise process and test if the residuals are normally distributed.
B. Assume the logarithm of the time series is a white noise process and test if the residuals are normally distributed.
C. Assume the first differences of the logarithm of the time series is a white noise process and test if the residuals are normally distributed.
D. Assume the second differences of the logarithm of the time series is a white noise process and test if the residuals are normally distributed.
E. Diagnostic tests are used if the error term is normally distributed; thay are not used if the error term in the original time series is lognormally distributed.

Answer 14.2: C

The first differences of the logarithms of the original time series would be a white noise process with a normally distributed error term.

*Question 14.3: Autocorrelation Significance

We are testing whether the observed autocorrelations of a time series are significant at the 5% level. We have 1,067 observations, and the critical *t* value at a 5% level of significance is 1.96. The null hypothesis is that the autocorrelations are zero.

A sample autocorrelation is significant at the 5% level of significance if its absolute value is larger than which of the following?

A. 0.011
B. 0.030
C. 0.022
D. 0.060
E. 0.090

Answer 14.3: D

If the *expected* autocorrelations are zero, the *observed* values (sample autocorrelations) are randomly distributed about zero. Their standard deviation is $1/\sqrt{T} = 1/1{,}067^{1/2} = 0.031$.

1.96 standard deviations is $1.96 \times 0.031 = 0.060$

Jacob: If an observed sample autocorrelation is greater (in absolute value) than 0.060, do we assume that the true autocorrelation is non-zero?

Rachel: If we examine 100 sample autocorrelations, we expect 5 of them to be greater (in absolute value) than 0.060. If 5 or fewer are greater than 0.060, we assume they are all zero, and these sample autocorrelations stem from fluctuations. Even if 7 or 8 of them are greater than 0.060, we might attribute this to chance. But if the first 3 autocorrelations are greater than 0.060 (especially if they are much greater than 0.060) and none of the subsequent sample autocorrelations are greater than 0.060, we might assume a MA(3) process.

*Question 14.4: Bartlett's test

A time series of 144 observations, t=1, 2, …, 144, has $\sum \hat{\varepsilon}_t$=0 and $\sum (y_t - \bar{y})(y_{t+k} - \bar{y})$ =

- 324 for k = 0
- 108 for k = 1
- 96 for k = 2
- 84 for k = 3
- 72 for k = 4
- 60 for k = 5
- 48 for k = 6
- 36 for k = 7
- 24 for k = 8
- 12 for k = 9

We are testing the null hypotheses $\rho_k = 0$ against the alternative hypotheses $\rho_k \neq 0$ using a separate test for each value of *k*. What is the lowest value of k for which we would not reject the null hypothesis at a 5% level of significance, for which the critical z-value is about 2?

A. 2
B. 4
C. 6
D. 8
E. 9

Answer 14.4: C

We find the *lowest* value of *k* for which the *z*-value is *less* than 2.

z = $\hat{\rho}_k \sqrt{T}$, where $\hat{\rho}_k = \dfrac{\sum (y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum (y_t - \bar{y})^2}$.

(W / 324) × $\sqrt{144}$ = 2 ⇛ W = 324 × 2 / 12 = 54.000
For k = 6, W = 48 < 54