TS Module 19 Seasonal models basics

(The attached PDF file has better formatting.)

Time series practice problems seasonal models

*Question 19.1: Seasonal moving average process

A time series is generated by $Y_t = e_t - e_{t-12}$.

What is ρ_{12}?

A. –1.0
B. –0.5
C. 0
D. +0.5
E. +1.0

Answer 19.1: B

The seasonal correlation is $-\Theta / (1+ \Theta^2) = -1 / (1 + 1^2) = -0.5$

(See Cryer and Chan, page 229: equation 10.1.3)

*Question 19.2: Seasonal moving average process

A seasonal moving average process has a characteristic polynomial of $(1 - \theta x)(1 - \Theta x^{12})$, with $\theta = 0.4$, $\Theta = 0.5$, and $\sigma^2_e = 4$

What is γ_0?

A. 0.20
B. 0.80
C. 1.60
D. 3.20
E. 5.80

Answer 19.2: E

(P230: equation 10.2.2: $\gamma_0 = (1 + \theta^2)(1 + \Theta^2)\, \sigma^2_e$)

$(1.16 \times 1.25 \times 4 = 5.80$

*Question 19.3: Seasonal moving average process

A seasonal moving average process has a characteristic polynomial of $(1 - \theta x)(1 - \Theta x^{12})$, with $\theta = 0.4$, $\Theta = 0.5$, and $\sigma^2_e = 4$

What is ρ_{12}?

A. –0.50
B. –0.40
C. 0
D. +0.40
E. +0.50

Answer 19.3: B

(P231: equation 10.2.5: $\rho_{12} = -\Theta / (1+ \Theta^2) = -0.5 / 1.25 = -0.4$

*Question 19.4: Seasonal ARMA process

A seasonal ARMA process $Y_t = \Phi\, Y_{t-12} + e_t - \theta\, e_{t-1}$ has $\Phi = -0.707$, $\theta = 1.414$, and $\sigma^2_e = 4$

What is γ_0?

A. 2
B. 4
C. 8
D. 16
E. 24

Answer 19.4: E

See Cryer and Chan, page 232, equation 10.2.11:

$$\gamma_0 = \sigma^2_e \times (1 + \theta^2) / (1 - \Phi^2) = 4 \times (1 + 2) / (1 - ½) = 24$$

The variance is the product of the seasonal autoregressive process and the non-seasonal moving average process.

*Question 19.5: Seasonal ARMA process

A seasonal ARMA process $Y_t = \Phi\, Y_{t-12} + e_t - \theta\, e_{t-1}$ has $\Phi = -0.707$, $\theta = 1.414$, and $\sigma^2_e = 4$

What is $\rho_{11} - \rho_{13}$?

A. –1.00
B. –0.50
C. 0
D. +0.50
E. +1.00

Answer 19.5: C

(See Cryer and Chan, page 232, equation 10.2.11: $\rho_{12k-1} = \rho_{12k+1} = -\Phi^k\, \theta / (1 + \theta^2)$)

For a stationary process, $\rho_1 = \rho_{-1}$. We form the autocorrelations for the 12 months seasonal lags, and then add or subtract one month for the non-seasonal lag.

*Question 19.6: Non-stationary seasonal ARIMA process

Suppose $Y_t = S_t + e_t$, with $\sigma^2_e = 2$ and $S_t = S_{t-s} + \epsilon_t$, with $\sigma^2_\varepsilon = 1$

What is the variance of $\nabla_s Y_t$?

A. 1
B. 2
C. 3
D. 4
E. 5

Answer 19.6: E

$\nabla_s Y_t = Y_t – Y_{t-s}$

$\Rightarrow \nabla_s Y_t = S_t – S_{t-s} + e_t – e_{t-s} = \epsilon_t + e_t – e_{t-s}$
$\Rightarrow$ variance $\nabla_s Y_t = 1 + 2 + 2 = 5$

*Question 19.7: Seasonal $AR(1)_{12}$ process

A store's toy sales in constant dollars follow a seasonal $AR(1)_{12}$ process: $Y_t = \Phi\, Y_{t-12} + e_t$.

- Sales are $10,000 in January 20X1 and $100,000 in December 20X1.
- Projected sales are $110,000 in December 20X2.

What are projected sales for January 20X3?

A. $10,000
B. $11,000
C. $11,100
D. $12,000
E. $12,100

Answer 19.7: E

- The December projection is one period ahead.
- The January projection is two periods ahead.

(See Cryer and Chan, page 241, last line; $\Phi = 1.1$)

Note that we take logarithms and first differences to make this process stationary.