

Macroeconomics, Module 3: “Growth Models” (Chapter 3)

Homework Assignment: Cobb-Douglas Production Function

(The attached PDF file has better formatting.)

Assume the economy has a Cobb-Douglas production function: $Y = AK^\alpha L^{(1-\alpha)}$. See Equation 3.23 on page 49 of the textbook (Part C of the Appendix to Chapter 3).

- A. What is the formula for capital per worker (k^*) in a steady state?
- B. What is the formula for output per worker (y^*) in a steady state?
- C. How does k^* change if the technology level A increases?
- D. How does k^* change if the savings rate s increases?
- E. How does k^* change if the depreciation rate δ increases?
- F. How does k^* change if the population growth rate n increases?
- G. (Optional:) What is the growth rate of capital per worker during the transition phase?

To solve this problem, write the Cobb-Douglas production function in a *per worker* form. Divide the function by L , the amount of labor. In a steady state, production per worker is constant and capital per worker is constant.

$$y = Y / L = AK^\alpha L^{(1-\alpha)} / L = A (K / L)^\alpha = A k^\alpha$$

(See Equation 3.24 on page 50).

Combine this equation with Equation 3.18 on page 46 (the box called “Do the Math”). Simplify this equation to get the value of k^* as a function of s , A , n , δ , and α (Part A).

You should get $k^* = [f(s, A, n, \delta)]^{1/(1-\alpha)}$, where f is a function of the four variables shown. For the homework assignment, solve for this function.

Having solved for k^* , use the relation above to solve for y^* (Part B). Raise k^* to the power of α and multiply by A .

For Parts C, D, E, F, use the expression for k^* . For the homework assignment, determine:

If (A , s , δ , or n) increases, does k^* increase or decrease?

Part G asks for $(\partial k)/k$. Use Equation 4.1 on page 51 for the growth rate of capital per worker as a function of s , y , n , and δ . You may post your solution on the discussion board to compare with others. See Equation 3.15 on page 42 and Equation 3.16 on page 43. See also the homework assignment for the next module, which examines the growth rate of capital during the transition phase. [Part G is optional for the homework assignment.]

Some final exam problems use the Cobb-Douglas production function and give α . Remember that α is the exponent of K , and $(1 - \alpha)$ is the exponent of L .