Macroeconomics, Module 5: Conditional Convergence and Long-run Economic Growth

Homework Assignment: Income Inequality

(The attached PDF file has better formatting.)

We examine absolute convergence with random fluctuations. This homework assignment leads you through the reasoning. The SOA and CAS exams use the stochastic models discussed here, so this exercise is useful for many applications.

- Suppose all countries have the same steady state income levels and growth rates.
- Current income differs by country because of random fluctuations in the past.

Random fluctuations continue to affect economic growth. One country has coal deposits, another country has oil, another country has natural gas, and another country has uranium. Each country's income depends on the technologies for producing energy and new finds of oil or natural gas. Solar power, wind power, and water power are other sources of energy which differ by country.

A friend tells you that if countries display absolute convergence, income inequality among the countries will decrease each year. Your friend argues that

- The countries have the same steady state income levels.
- If Country S has a higher than average income level now, it grows more slowly, and if Country T has a lower than average income level now, it grows faster.
- Poorer countries grow faster than richer countries, so income inequality decreases.

Part A: Explain why this argument is incorrect.

Barro refers to this as Galton's fallacy. The original form of the argument is

- Tall fathers tend to have sons who are shorter than they are (on average).
- Short fathers tend to have sons who are taller than they are (on average).

One might suppose that after a few generations, all men will have the same height. But the variance of men's height is not decreasing. That is, mean reversion does not decrease the variance of men's height in the population.

Part B: Part B of the homework assignment examines income inequality in a steady state. The solution is a simple formula; the homework assignment derives this formula.

- Suppose all countries have a steady state income of $10,000 per capita.
- The mean reversion is 20%. If a country's income this year is Y, its expected income next year is 20% × $10,000 + 80% × Y.
- Each country's income has a random factor of Z. If its expected income next year is W (per capita), its *actual* income is W + Z (per capita).

- Z has a normal distribution with a mean of zero and a standard deviation of $1,000.
- Assume the random fluctuations each year are *independent*. A high value of Z one year does not imply a high value the next year.

In a steady state, what is the standard deviation of income per capita among countries?

Use the following reasoning to solve this homework assignment. The mathematics is

- If A and B are independent random variables, Var(A + B) = Var(A) + Var(B).
- Var(kA) = k^2 × Var(A), where *k* is a constant.

We consider first the two end points:

- if the mean reversion is 100%, income inequality stems only from the random fluctuations that year, so the standard deviation is $1,000.
- If the mean reversion is 0%, the variance in year T+1 = the variance in year T + Var(Z). The variance (and the standard deviation) increases each year.

We examine mean reversion:

If the standard deviation of the current year's income is σ, its variance is σ^2. The variance of next year's income has two pieces:

- A reduction of the variance caused by the mean reversion. If the mean reversion is P%, the variance next year is $(1 - P\%)^2 \times \sigma^2$.
- A variance of $\$1{,}000^2$ caused by the random fluctuations (the stochastic term Z).

These two pieces are independent and can be added. In a steady state, the sum is σ^2. For a given mean reversion, we solve for a steady state σ^2. For the homework assignment, use a mean reversion of 20% to solve for the steady state σ^2.

By the way, this is the variance of a stationary autoregressive process, discussed in the time series on-line course.