Macroeconomics, Module 12: Demand for Money

Homework Assignment: Demand for Money

(The attached PDF file has better formatting.)

This homework assignment derives the demand for money by maximizing a person's net wealth from transaction costs of bank withdrawals and the interest income on bonds. The scenario is simplistic. Real world scenarios are more complex, but the reasoning is the same.

- A person is paid $100,000 on January 1 by direct deposit into a savings account. The person does not have to visit the bank to deposit the money.
- The person spends $100,000 during the year.
- Interest of 10% per annum is paid on the money in the savings account.

Money is withdrawn from the savings account by a visit to a branch office of the bank, which takes half an hour. The person values time at $100 an hour, so the transaction costs of the bank visit are $50.

The person withdraws money N times a year. We find the value of N that maximizes the person's wealth (minimizes transaction costs minus loss of interest).

We first examine several scenarios to show that N lies between 2 and 250. We compute the net cost if the person withdraws money from the savings account

- Once a year
- Twice a year.
- Every business day, or 250 times a year.

We then solve for withdrawals of N times a year, where N maximizes the person's wealth, or minimizes the net cost of holding cash.

A. If the person withdraws money once a year (on January 1), what are the annual transaction costs and the annual interest earned?
B. We show that a single withdrawal is not optimal. If the person withdraws money twice a year – January 1 and July 1 – what are the annual transaction costs and the annual interest earned?
C. Two many transactions are not optimal. If the person withdraws money 250 times a year, what are the annual transaction costs and the annual interest earned?
D. If the person withdraws money N times a year, what are the annual transaction costs and the annual interest earned? (Transaction costs = $100 × N, interest = (N-1)/(2N) × $100,000 × 10%)

For Part A, the person incurs a single transaction cost, but nothing is left in the savings account.

For Part B, the person incurs two transaction costs. Work out the average amount in the savings account during the year to determine the interest revenue. The person withdraws $50,000 on January 1 and $50,000 on July 1.

For Part C, you need not work out the exact answer. The transaction costs are 250 × $50. Estimate the interest if withdrawals are continuous. With continuous withdrawals, the average account balance is ½ × $100,000.

For Part D, the transaction costs are $50 × N. To compute the average account balance with N withdrawals, with the first on January 1, divide the year into N segments. N may be odd or even; the formula is the same.

- In segment 1, the account balance is (N-1)/N × $100,000.
- In segment 2, the account balance is (N-2)/N × $100,000.
- …
- In segment (N-1), the account balance is 1/N × $100,000.
- In segment N, the account balance is 0/N × $100,000.

The segments all have the same length.

- The average account balance in segment 1 and segment N is ½ × (N-1)/N × $100,000.
- The average account balance in segment 2 and segment N-1 is ½ × (N-1)/N × $100,000.

We infer that the average account balance during the year is (N-1)/(2N) × $100,000.

We must maximize (½ – 1/2N)× $10,000 – $50N. This is the same as minimizng 5,000/N + 50N. Set the partial derivative with respect to N equal to zero.