Macroeconomics, Module 18: Taxes

Homework Assignment: Marginal Tax Rates and Labor Supply

(The attached PDF file has better formatting.)

This homework assignment examines marginal tax rates, flat tax rates, and the effects on labor supply. The assumptions about the number of workers and distribution of pre-tax income simplify the mathematics; they are not essential to the concepts. Assume:

- Country W has 100,000 workers.
- Annual pre-tax income in Country W is uniformly distributed over ($0, $100,000).
- The current tax rate is 20% of all income.

To reduce income inequality, the government changes the tax rate to 0% at income of $0 rising linearly to 30% at income of $100,000.

A. What is the total tax revenue before the change in the tax rate?
B. What is the total tax revenue after the change in the tax rate if people do not change their hours of work?
C. What is the marginal tax rate after the change in the tax rate?
D. What is the tax rate and the marginal tax rate for a person earning $40,000 a year?
E. If the marginal tax rate affects labor supply (as Barro assumes), will pre-tax income rise or fall after the change in the tax rate?
F. An economist recommends that the tax rate be changed to 0% on the first $20,000 of income and 31.25% on income above $20,000. If people do not change their hours of work, what is the total tax revenue?
G. What is the marginal tax rate for the economist's recommendation?
H. If marginal tax rates affect hours worked, explain why this recommendation solves the income inequality problem but is better than the government's change.

To solve this homework assignment, let Y be pre-tax income in units of $100,000. The *pdf* (probability density function) of Y is f(y) = 1 as Y ranges from 0 to 1. Units of $100,000 eliminate the zeros from the computations.

$\tau(y)$ is the tax rate. The tax liability is $\tau(y) \times y$. Note that $\tau(y)$ is the average tax rate for a given income level, not the marginal tax rate at that income level. You work out the marginal tax rate to solve the homework assignment.

For Parts A, B, and F: The total tax revenue for the population is the tax liability for each income level times the distribution of income levels in the population, or the integral of $y \times f(y) \times \tau(y)$ from 0 to 1.

$$\int_0^1 y \times f(y) \times \tau(y) dy$$

The tax rate $\tau(y)$ is

- 20% for all y before the change.
- 30% × *y* after the change (*y* is in units of $100,000). The tax liability is 30% × y × y.
- For the economist's recommendation: 0% for $y < 0.2$ ($20,000), and 31.25% applied to $(y - 0.2)$ for $y > 0.2$ ($20,000).

For Part C, the marginal tax rate is the derivative of the tax revenue at a given income level with respect to income. If the tax rate at income level *y* is $\tau(y)$, the marginal tax rate at an income level *y* is $\partial(y \times \tau(y))/\partial y$. If $\tau(y) = \alpha \times y$, the marginal tax rate is $2 \times \alpha \times y$. [This relation is asked on the final exam as well.]

For Part D, use $y = 0.400$ in the formula you derive for Part C. *To check your answer, derive the tax liability at income levels of $40,000 and $40,001. The difference in the tax liabilities is the marginal tax rate at an income level of $40,000.*

Note that the tax rate decreased for this worker after the change but the marginal tax rate increased. The total tax paid is lower, but the worker has *less incentive to work*.

For Part E, use two characteristics of high vs low-paid workers.

- Low paid employees work all day to make ends meet. Reducing their taxes has moral justification, but it won't induce them to work more. The moral justification is that if a person works all day to buy food and clothing, taxing the person's income causes more pain than the benefits that person gets from government services. Whether this is true depends on the type of services that government provides.
- High paid workers balance more consumption vs more leisure. If their marginal tax rate increases, they work less. The moral justification for progressive tax rates depends on the type of services that government provides.

Show that the income level at which the marginal tax rate increases is $y = \frac{1}{3}$, or $33,333.

- The total tax revenue does not change.
- Two thirds of workers have lower *average* tax rates.
- Two thirds of workers have higher *marginal* tax rates.

For Part F: If a person earns $50,000, the tax liability is

$$0\% \times \$20{,}000 + 31.25\% \times (\$50{,}000 - \$20{,}000).$$

For the total tax revenue, integrate $31.25\% \times (y - 0.2)\, dy$ from $y = 0.2$ to $y = 1.0$. The *pdf* of y is still $f(y) = 1$.

For Part G: The economist's proposal is a combination of two flat tax rates: one on the first $20,000 of income and the other on the next $80,000 of income. The marginal tax rate is one of two values, depending on the income of the worker.

For Part H, compare the marginal tax rates for the three tax systems.

- Low paid workers pay less tax with the economist's recommendation, and their marginal tax rates are lower.
- Many high paid persons pay more tax with the economist's recommendation, but their marginal tax rates are lower.

State which persons have higher marginal tax rates for each tax system. Explain how this affects the incentive to work.