

Time series mod 5: MA(2) autocorrelations practice problems

For AR(1), AR(2), MA(1), MA(2), and ARMA(1,1) processes, know how to calculate γ_0 , γ_1 , γ_2 , ρ_1 , and ρ_2 from ϕ_1 , ϕ_2 , θ_1 , θ_2 , and σ_ε . Know the formulas for these five simple processes. For more complex processes, such as ARMA(p, q) for $p > 1$ and $q > 1$, or AR(p) for $p > 2$, or MA(q) for $q > 2$, know the principles, such as the shapes of the sample autocorrelation and partial autocorrelation functions.

Exercise 1.1: MA(2) process

An MA(2) process has $\theta_1 = 0.7$, $\theta_2 = 0.5$, and $\sigma_\varepsilon = 2$.

- A. What is γ_0 ?
- B. What is γ_1 ?
- C. What is γ_2 ?
- D. What is ρ_1 ?
- E. What is ρ_2 ?

Solution 1.1: For an MA(2) process:

$$\gamma_0 = (1 + \theta_1^2 + \theta_2^2) \times \sigma^2$$

$$\gamma_1 = (-\theta_1 + \theta_1 \times \theta_2) \times \sigma^2$$

$$\gamma_2 = (-\theta_2) \times \sigma^2$$

$$\rho_1 = (-\theta_1 + \theta_1 \times \theta_2) / (1 + \theta_1^2 + \theta_2^2)$$

$$\rho_2 = (-\theta_2) / (1 + \theta_1^2 + \theta_2^2)$$

$$\rho_k = 0 \text{ for } k = 3, 4, \dots$$

Cryer and Chan, page 63 (equation 4.2.3)

$$\text{Part A: } \gamma_0 = (1 + \theta_1^2 + \theta_2^2) \times \sigma^2 = (1 + 0.49 + 0.25) \times 2^2 = 6.960$$

$$\text{Part B: } \gamma_1 = (-\theta_1 + \theta_1 \cdot \theta_2) \times \sigma^2 = (-0.7 + 0.7 \times 0.5) \times 2^2 = -1.400$$

$$\text{Part C: } \gamma_2 = (-\theta_2) \times \sigma^2 = -0.5 \times 2^2 = -2.000$$

$$\text{Part D: } \rho_1 = (-\theta_1 + \theta_1 \times \theta_2) / (1 + \theta_1^2 + \theta_2^2) = -0.201$$

$$\text{Part E: } \rho_2 = (-\theta_2) / (1 + \theta_1^2 + \theta_2^2) = -0.287$$