TS module 12: Method of moments for ARMA(1,1) process (practice problem)

(The attached PDF file has better formatting.)

Know how to estimate ϕ and θ for an ARMA(1,1) process by the method of moments. You solve a quadratic equation for θ.

Exercise 1.2: ARMA(1,1) model and method of moments (Yule-Walker equations)

An ARMA(1,1) model is fit to a time series with sample autocorrelations for the first two lags of $r_1 = 0.880$ and $r_2 = 0.704$.

A. What is the method of moments estimate for ϕ?
B. What is the method of moments estimate for θ?

Part A: For an ARMA(1,1) process, $r_2 = r_1 \times \phi \Rightarrow \phi = 0.704 / 0.880 = 0.8$

Part B: For an ARMA(1,1) process (for $k \geq 1$): $\rho_k = \dfrac{(1-\theta\phi)(\phi-\theta)}{1-2\theta\phi+\theta^2}\phi^{k-1}$

We estimated ϕ as r_2 / r_1. We estimate θ from $r_1 = \dfrac{(1-\theta\hat{\phi})(\hat{\phi}-\theta)}{1-2\theta\hat{\phi}+\theta^2}$

See Cryer and Chan, equation 7.1.6 on page 151.

In this exercise, $0.880 = (1 - 0.8\,\theta)(0.8 - \theta) / (1 - 2(0.8\,\theta) + \theta^2)$.

This is a quadratic equation in θ, with roots of -0.4 and -2.5 (use the formula for roots of a quadratic).

The arithmetic is shown below; most final exam problems use simple numbers.

$0.880 = (1 - 0.8\,\theta)(0.8 - \theta) / (1 - 2(0.8\,\theta) + \theta^2)$

$0.880 \times (1 - 2(0.8\,\theta) + \theta^2) = (1 - 0.8\,\theta)(0.8 - \theta)$

$88 \times (1 - 2(0.8\,\theta) + \theta^2) = (10 - 8\,\theta)(8 - 10\,\theta)$

$88 - 140.8\,\theta + 88\,\theta^2 = 80 - 164\,\theta + 80\,\theta^2$

$8 + 23.2\,\theta + 8\,\theta^2 = 0$

Using the formula for the roots of a quadratic equation gives

$(8^2 \pm (23.2^2 - 4 \times 8 \times 8))\,/\,(2 \times 8) = -0.4$ and -2.5