TS module 12 method of moments practice problem

Estimates of μ, γ_0, and ρ do not depend on the ARIMA process.

- Estimate μ from the mean of the observations.
- Estimate γ_0 from the variance of the observations.
- Estimate ρ from the sample autocorrelations.

Exercise 12.1: The first five values of a stationary time series are 6, 5, 4, 6, and 4.

A. What is the estimate of μ?
B. What is the estimate of γ_0?
C. What is the estimate of ρ_1?

Part A: An unbiased estimator of μ is $\sum Y_t / N = (6 + 5 + 4 + 6 + 4) / 5 = 25/5 = 5$.

Part B: An unbiased estimator of γ_0 is the variance of the observed values =

$$(1^2 + 0^2 + (-1)^2 + 1^2 + (-1)^2) / 4 = 1.$$

Part C: The numerator of the sample autocorrelation is

$$(1 \times 0 + 0 \times -1 + -1 \times 1 + 1 \times -1) = -2.$$

The denominator is $\gamma_0 = 4$ (see Part B), so $\rho_1 = -2/4 = -½$.

Jacob: Why doesn't this exercise ask for σ^2_ε?

Rachel: The estimate of σ^2_ε depends on the type of model, such as AR(1), MA(1), or ARMA(1,1).

Jacob: Can't we select the best model?

Rachel: With only five observed values, the standard error of the observed sample autocorrelations is high: $1/\sqrt{5} = 0.44721$. The width of the 95% confidence interval is $2 \times 1.96 \times 0.44721 = 1.75308$. The data are too sparse to select the best model.