TS module 16 Random walk forecasting

(The attached PDF file has better formatting.)

Cryer and Chan, chapter 9, pages 198-199, discuss forecasting of random walks.

Exercise 16.1: White Noise Process and Random Walk

- A white noise process has parameters $\mu = 1$ and $\sigma^2_\varepsilon = 4$.
  - A white noise process is an MA(1) process with $\theta_1 = 0$.
- A random walk with drift = 1 and $\sigma^2_\varepsilon = 4$ is the cumulative sum of the white noise terms.
- The most recent value of the random walk is 10.

A. What are the one- and two period ahead forecasts of the white noise process?
B. What are the one- and two period ahead forecasts of the random walk?
C. What are the variances of the one- and two period ahead forecasts of the white noise process?
D. What are the variances of the one- and two period ahead forecasts of the random walk?

Part A: A white noise process has no serial correlation. Each value is independent of the other values. The expected value (forecast) is the mean μ (= 1 in this exercise).

Take heed: In earlier chapters, Cryer and Chan often assume the mean of the white noise process is zero. The residuals from a correctly specified model are a white noise process with a mean of zero. In practice, random walks often have drifts $\Rrightarrow$ the first differences are white noise processes with $\mu \neq 0$.

Part B: The random walk is the cumulative sum of the white noise process. The most recent value is 10, so the one period ahead forecast is 11, the two periods ahead forecast is 12, and so forth.

Part C: Each value of the white noise process is $\mu + \epsilon_t$. The mean μ is a scalar with no variance, and the residual ϵ_t is a random variable with a variance of $\sigma^2_\varepsilon = 4$, so each forecast has a variance of 4.

Part D: Each value of the random walk is $\sum\mu + \sum\epsilon_t$. The mean μ is a scalar with no variance, so the cumulative sum of the scalars also has no variance.

- The error term in a future value is a random variable with a variance of σ^2_ε.
- The residual in an observed value is a realization of the random variable, which is a scalar with no variance.
- The error terms are independent with a variance of $\sigma^2_\varepsilon = 4$ for each one.
- The variance of a sum of independent random variables is the sum of the variances.
- The variance of the *k* period ahead forecast k × σ^2_ε.