TSS module 9 Identifying ARIMA processes.

Cryer and Chan show how to write an ARIMA(p,1,q) process as a non-stationary ARMA process. Some final exam problems ask you to identify the proper ARIMA process and its parameters.

**Question 1.2: ARIMA Process

A time series is $Y_t = 1.4Y_{t-1} + 0.1Y_{t-2} – 0.5Y_{t-3} + e_t + 0.3e_{t-1} + 0.2e_{t-2}$

What is the process followed by this time series?

A. ARIMA(1,1,1)
B. ARIMA(2,1,2)
C. ARIMA(2,1,1)
D. ARIMA(1,2,1)
E. ARIMA(2,2,1)

Answer 1.2: B

Rewrite the ARIMA process as

$Y_t – Y_{t-1} = 0.4Y_{t-1} + 0.1Y_{t-2} – 0.5Y_{t-3} + e_t + 0.3e_{t-1} + 0.2e_{t-2}$

$= 0.4Y_{t-1} – 0.4Y_{t-2} + 0.4Y_{t-2} + 0.1Y_{t-2} – 0.5Y_{t-3} + e_t + 0.3e_{t-1} + 0.2e_{t-2}$

$= 0.4Y_{t-1} – 0.4Y_{t-2} + 0.5Y_{t-2} – 0.5Y_{t-3} + e_t + 0.3e_{t-1} + 0.2e_{t-2}$

$\Rightarrow W_t = 0.4 W_{t-1} + 0.5W_{t-2} + e_t + 0.3e_{t-1} + 0.2e_{t-2}$

See equation 5.2.2 on page 92.

Intuition: The ARIMA(p,1,q) process is

$Y_t – Y_{t-1} = \phi_1 (Y_{t-1} – Y_{t-2}) + \phi_2 (Y_{t-2} – Y_{t-3}) + \ldots + \phi_p (Y_{t-p} – Y_{t-p-1}) + \epsilon_t – \theta_1 \epsilon_{t-1} – \theta_2 \epsilon_{t=2} – \ldots – \theta_q \epsilon_{t-q}$

Rewrite this as

$Y_t = (1 + \phi_1) Y_{t-1} + (\phi_2 – \phi_1) Y_{t-2} + (\phi_3 – \phi_2) Y_{t-3} + \ldots + (\phi_p – \phi_{p-1}) Y_{t-p} + + \epsilon_t – \theta_1 \epsilon_{t-1} – \theta_2 \epsilon_{t=2} – \ldots – \theta_q \epsilon_{t-q}$

The AR(1) process for $\nabla Y_t = W_t$ has $\phi_1 = 0.4$ and $\phi_2 = 0.5$, which give

- $1 + \phi_1 = 1.4$
- $\phi_2 – \phi_1 = 0.1$
- $–\phi_2 = –0.5$

These are the coefficients of Y_{t-k} in the original equation.

Jacob: What about the moving average terms?

Jacob: The coefficients of the error terms ϵ_{t-k} remain unchanged; they are the negatives of the moving average parameters.

Jacob: What is the procedure for this transformation?

Rachel: The sum of the coefficients for the Y terms are equal on both sides of the equation.