TSS module 9 Time series differencing

Cryer and Chan show how differencing converts a non-stationary ARIMA process to a stationary time series.

** Question 1.2: Linear over three time points

Suppose that $Y_t = M_t + e_t$ with $M_t = M_{t-1} + \epsilon_t$

The time series M(t) is linear over *three* consecutive points, so the least squares estimator of M(t) is ⅓ $(Y_{t-1} + Y_t + Y_{t+1})$

Which of the following is stationary?

A. Y(t)
B. ∇Y(t)
C. ∇^2Y(t)
D. ⅓ ∑Y(t)
E. Y(t) – Y(t-1) + Y(t-2)

Answer 1.2: C

If M_t is linear over three points, the best estimate for M_t is the centered moving average of three Y_t values:

$$\hat{M}_t = \tfrac{1}{3}\left(Y_{t+1} + Y_t + Y_{t-1}\right)$$

If we remove the trend, the time series is stationary.

$$Y_t - \hat{M}_t = Y_t - \tfrac{1}{3}\left(Y_{t+1} + Y_t + Y_{t-1}\right)$$

Simplify the right hand side of this equation:

$= -\tfrac{1}{3}\,(Y_{t+1} - 2\,Y_t + Y_{t-1})$

$= -\tfrac{1}{3}\,\nabla(\nabla Y_{t+1})$

$= -\tfrac{1}{3}\,\nabla^2(Y_{t+1})$

See Cryer and Chan, chapter 5, page 91

**Question 1.3: First difference

Suppose $Y_t = M_t + e_t$ and $M_t = M_{t-1} + \epsilon_t$

What is ∇Y_t (the first difference of Y_t)?

A. ϵ_t
B. e_t
C. $\epsilon_t - e_t - e_{t-1}$
D. $\epsilon_t + e_t - e_{t-1}$
E. $\epsilon_t + e_t + e_{t-1}$

Answer 1.3: D

(See Cryer and Chan, chapter 5, page 90, Equation 5.1.9)

$\nabla Y_t = Y_t - Y_{t-1} = M_t + e_t - (M_{t-1} + e_{t-1}) = \epsilon_t + e_t - e_{t-1}$

Final exam problems give $Y_t = \boldsymbol{k} \times M_t + e_t$ (where k is a scalar) and the variances of e_i and ϵ_t. You must work out variances, auto-covariances, and autocorrelations.

**Question 1.4: ARIMA(p,1,q) process

An ARIMA(p,1,q) process Y_t has first differences ∇Y_t that are an ARMA(p,q) process with parameters ϕ_i and θ_j.

The ARIMA(p,1,q) process is written as a *non-stationary* moving average process ARMA(p+1, q).

What is the coefficient of Y_{t-2} in the non-stationary ARMA(p+1, q) process?

A. $1 - \phi_2$
B. $\phi_1 - \phi_2$
C. $\phi_2 - \phi_1$
Δ. $\phi_2 - 1$
E. $1 - \phi_1 - \phi_2$

Answer 1.4: C

See equation 5.2.2 on page 92.

Intuition: The ARIMA(p,q) process is

$$Y_t - Y_{t-1} = \phi_1 (Y_{t-1} - Y_{t-2}) + \phi_2 (Y_{t-2} - Y_{t-3}) + \ldots + \phi_p (Y_{t-p} - Y_{t-p-1}) + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t=2} - \ldots - \theta_q \epsilon_{t-q}$$

Rewrite this as

$$Y_t = (1 + \phi_1) Y_{t-1} + (\phi_2 - \phi_1) Y_{t-2} + (\phi_3 - \phi_2) Y_{t-3} + \ldots + (\phi_p - \phi_{p-1}) Y_{t-p} + + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t=2} - \ldots - \theta_q \epsilon_{t-q}$$

**Question 1.5: ARIMA(p,1,q) process

An ARIMA(p,1,q) process Y_t has first differences ∇Y_t that are an ARMA(p,q) process with parameters ϕ_i and θ_j.

The ARIMA(p,1,q) process is written as a *non-stationary* moving average process ARMA(p+1, q).

What is the coefficient of ϵ_{t-2} in the non-stationary ARMA(p+1, q) process?

A. $-\theta_2$
B. $1 - \theta_2$
C. $\theta_1 - \theta_2$
D. $\theta_2 - \theta_1$
E. $\theta_2 - 1$

Answer 1.5: A

See equation 5.2.2 on page 92.

Intuition: The ARIMA(p,q) process is

$$Y_t - Y_{t-1} = \phi_1 (Y_{t-1} - Y_{t-2}) + \phi_2 (Y_{t-2} - Y_{t-3}) + \dots + \phi_p (Y_{t-p} - Y_{t-p-1}) + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t=2} - \dots - \theta_q \epsilon_{t-q}$$

Rewrite this as

$$Y_t = (1 + \phi_1) Y_{t-1} + (\phi_2 - \phi_1) Y_{t-2} + (\phi_3 - \phi_2) Y_{t-3} + \dots + (\phi_p - \phi_{p-1}) Y_{t-p} + + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t=2} - \dots - \theta_q \epsilon_{t-q}$$

The coefficients of the error terms ϵ_{t-k} remain unchanged; they are the negatives of the moving average parameters.