TSS module 9 ARI processes.

Cryer and Chan, chapter 5, show how to convert ARI (intergrated autoregressive) processes to stationary autoregressive process by taking differences. The ARI(1,1) format is shown in this illustrative test question, using equation 5.2.12 on page 97. Cryer and Chan provide the general expression for ARI(p,1) processes.

*Question 1.2: ARI(1,1) process

Which of the following is an ARI(1,1) process?

A. $Y_t = Y_{t-1} - (1 + \phi)\, Y_{t-2} + e_t$
B. $Y_t = Y_{t-1} + \phi\, Y_{t-2} + e_t$
C. $Y_t = Y_{t-1} - \phi\, Y_{t-2} + e_t$
D. $Y_t = (1 + \phi)\, Y_{t-1} - \phi\, Y_{t-2} + e_t$
E. $Y_t = (1 + \phi)\, Y_{t-1} + \phi\, Y_{t-2} + e_t$

Answer 1.2: D

$Y_t = (1 + \phi)\, Y_{t-1} - \phi\, Y_{t-2} + e_t$

$\Rightarrow Y_t - Y_{t-1} = \phi\, Y_{t-1} - \phi\, Y_{t-2} + e_t$

$\Rightarrow \nabla Y_t = \phi\, \nabla Y_{t-1} + e_t$

Final exam problems give ϕ_1, ϕ_2, … and derive the underlying AR(p) process. They also test variances of Y_t given a starting point for the time series, covariances, and correlations.

**Question 1.3: ARI(1,1) process

An ARI(1,1) process Y_t has first differences ∇Y_t that are an AR(1) process with parameter $\phi = 0.5$.

$Y_t - Y_{t-1} = \phi (Y_{t-1} - Y_{t-2}) + \epsilon_t$

The ARI(1,1) process is written as a non-stationary ARMA process with weights ψj applied to the error terms:

$Y_t = \psi_0 \epsilon_t + \psi_1 \epsilon_{t-1} + \psi_2 \epsilon_{t-3} + \psi_3 \epsilon_{t-3} + \ldots$

What is the value of ψ_2?

A. 0.25
B. 0.5
C. 0.75
D. 1
E. 1.75

Answer 1.3: E

See equation 5.2.15 on page 97:

$$\psi_k = \frac{1-\phi^{k+1}}{1-\phi} \; for \; k \geq 1$$

For $\phi = 0.5$ and k = 2, this gives $(1 - 0.5^{2+1}) / (1 - 0.5)$ = ⅞ / ½ = 1.75

Intuition: Expand the expression for the ARI(1,1) process:

$Y_t - Y_{t-1} = \phi (Y_{t-1} - Y_{t-2}) + \epsilon_t$

$Y_t = (1 + \phi) Y_{t-1} - \phi Y_{t-2} + \epsilon_t$

$Y_t = (1 + \phi) Y_{t-1} - \phi Y_{t-2} + \epsilon_t$