General linear processes practice problems

Module 6: Autoregressive processes

(The attached PDF file has better formatting.)

**Exercise 6.1: Geometric decay

A time series has the form $Y_t = \epsilon_t + \phi \times \epsilon_{t-1} + \phi^2 \times \epsilon_{t-2} + \phi^3 \times \epsilon_{t-3} + \ldots$

$\phi = 0.4$ and $\sigma^2_e = 4$.

A. What is γ_0, the variance of Y_t?
B. What is γ_1, the covariance of Y_t and Y_{t-1}?
C. What is ρ_1, the autocorrelation of Y_t and Y_{t-1}?
D. What is ρ_2, the correlation of Y_t and Y_{t-2}?

Part A: See Cryer and Chan, chapter 4, top of page 56:

$$\gamma_0 = \sigma^2 / (1 - \phi^2) = 4 / (1 - 0.16) = 4.762$$

Later modules refer to this process as AR(1), an autoregressive process of order 1. Final exam problems say: an AR(1) process with $\phi = 0.4$ and $\sigma^2_\varepsilon = 4$.

Part B: See Cryer and Chan, chapter 4, middle of page 56:

$$\gamma_1 = \phi \times \sigma^2 / (1 - \phi^2) = 0.4 \times 4 / (1 - 0.16) = 1.905$$

Part C: See Cryer and Chan, chapter 4, equation 4.1.3 at the bottom of page 56:

$$\rho_1 = \phi = 0.4$$

Part D: See Cryer and Chan, chapter 4, equation 4.1.3 at the bottom of page 56:

$$\gamma_2 = \phi^2 = 0.4^2 = 0.160$$

This exercise is simple. Final exam problems are more complex. The autoregressive process may be of an order higher than 1, the ϕ parameters may be positive or negative, the process may have a moving average part, and the parameters may be estimated from the observed sample autocorrelations. The logic is the same for all the scenarios. This exercise is a good starting point for stationary time series.