TS module 7 autocovariance and autocorrelations practice problems

(The attached PDF file has better formatting.)

For AR(1), AR(2), MA(1), MA(2), and ARMA(1,1) processes, know how to calculate γ_0, γ_1, γ_2, ρ_1, and ρ_2 from ϕ_1, ϕ_2, θ_1, and θ_2.

** Exercise 7.1: MA(2) process

An MA(2) process has $\theta_1 = 0.7$, $\theta_2 = 0.5$, and $\sigma_\varepsilon = 2$.

A. What is γ_0?
B. What is γ_1?
C. What is γ_2?
D. What is ρ_1?
E. What is ρ_2?

For an MA(2) process:

$\gamma_0 = (1 + \theta_1^2 + \theta_2^2) \times \sigma^2$
$\gamma_1 = (-\theta_1 + \theta_1 \times \theta_2) \times \sigma^2$
$\gamma_2 = (-\theta_2) \times \sigma^2$

$\rho_1 = (-\theta_1 + \theta_1 \times \theta_2) / (1 + \theta_1^2 + \theta_2^2)$
$\rho_2 = (-\theta_2) / (1 + \theta_1^2 + \theta_2^2)$
$\rho_k = 0$ for $k = 3, 4, \ldots$

See Cryer and Chan, page 63 (equation 4.2.3)

Part A: $\gamma_0 = (1 + \theta_1^2 + \theta_2^2) \times \sigma^2 = (1 + 0.49 + 0.25) \times 2^2 = 6.960$

Part B: $\gamma_1 = (-\theta_1 + \theta_1 \cdot \theta_2) \times \sigma^2 = (-0.7 + 0.7 \times 0.5) \times 2^2 = -1.400$

Part C: $\gamma_2 = (-\theta_2) \times \sigma^2 = -0.5 \times 2^2 = -2.000$

Part D: $\rho_1 = (-\theta_1 + \theta_1 \times \theta_2) / (1 + \theta_1^2 + \theta_2^2) = -0.201$

Part E: $\rho_2 = (-\theta_2) / (1 + \theta_1^2 + \theta_2^2) = -0.287$

** Exercise 7.2: Covariance of AR(1) process

An AR(1) process has an autoregressive parameter ϕ = 0.6 and σ_ε = 4.

A. What is the covariance of Y_t with Y_{t-1}? (1 period lag)
B. What is the covariance of Y_t with Y_{t-2}? (2 period lag)

Part A: See Cryer and Chan, page 66, equation 4.3.3:

$$\gamma_o = \frac{\sigma_\varepsilon^2}{1-\phi^2}$$

γ_0 is the variance of the time series elements. It is greater than σ^2_ε because the time series observations are autocorrelated. If an observation is high (low) one period, it is similarly high (low) the next period, instead of reverting to the mean. An autoregressive parameter ϕ closer to one causes higher autocorrelation and a higher variance for the time series.

The autocovariance (and autocorrelation) decline by exponential decay; see page 67, equation 4.3.5:

$$\gamma_k = \phi^k \frac{\sigma_\varepsilon^2}{1-\phi^2}$$

The covariance of Y_t with Y_{t-1} (1 period lag) is γ_1.

= $0.6 \times 4^2 / (1 - 0.6^2) = 15$

Part B: The covariance of Y_t with Y_{t-2} (2 period lag) is γ_2.

= $0.6^2 \times 4^2 / (1 - 0.6^2) = 9$

** Question 7.3: Standard deviation

If σ (the standard deviation of the time series) doubles, what is the effect on γ_1 and ρ_1?

A. γ_1 doubles, ρ_1 doesn't change
B. γ_1 quadruples, ρ_1 doesn't change
C. γ_1 doubles, ρ_1 doubles
D. γ_1 quadruples, ρ_1 doubles
E. γ_1 doesn't change, ρ_1 doesn't change

Answer 7.3: B

See Cryer and Chan, page 66, equation 4.3.3:

$$\gamma_o = \frac{\sigma_e^2}{1-\phi^2}$$

and page 67, equation 4.3.5:

$$\gamma_k = \phi^k \frac{\sigma_e^2}{1-\phi^2}$$

If σ doubles, all the γ terms quadruple. The autocorrelations (the ρ terms) are γ_k / γ_0, so they do not change.

Equation 4.3.5 gives the effect of ϕ and σ on the γ terms. Know this equation.