

Fox Module 17: Hat values practice problem

****Exercise 17.1: Hat values**

A statistician regresses the nine Y values on the nine X values.

Y	6.87	6.58	7.69	6.96	7.39	14.14	7.90	15.62	13.23
X	1	2	3	4	5	6	7	8	9

- A. What are the hat values at each point?
- B. What is the minimum hat value in any regression equation?
- C. What is the maximum hat value in any regression equation?

Part A: The hat values are shown in the table below.

# Pts	9	Deviance	Deviance Squared	Hat Value
values	1	-4	16	0.378
	2	-3	9	0.261
	3	-2	4	0.178
	4	-1	1	0.128
	5	0	0	0.111
	6	1	1	0.128
	7	2	4	0.178
	8	3	9	0.261
	9	4	16	0.378
ave/tot	5	0	60	0.222

Each hat value is: (Fox page 245)
$$h_i = \frac{1}{n} + \frac{(x_i - \bar{x})^2}{\sum_{j=1}^n (x_j - \bar{x})^2}$$

Part B: If $x_j = \bar{x}$, the hat value is $1/n$; this is the minimum.

Part C: Suppose the explanatory variable has N independent points, of which (N-1) are zero and 1 is N.

The points $x_j = 0$ have hat values of $1/n + 1/n^2 / ((n-1)/n^2 + (n-1)^2/n^2) = 1/n + 1 / ((n-1) + (n-1)^2)$.

As $n \rightarrow \infty$, the hat values $\rightarrow 1/n$.

The point $x_j = N$ has a hat value of $1/n + (n-1)^2/n^2 / ((n-1)/n^2 + (n-1)^2/n^2) = 1/n + (n-1)^2 / ((n-1) + (n-1)^2)$
 $= 1/n + (n-1) / n = 1$.

As $n \rightarrow \infty$, the hat value $\rightarrow 1$.