TS module 12 AR(2) process and method of moments (practice problem)

(The attached PDF file has better formatting.)

*Exercise 12.1: AR(2) process and method of moments

The first two sample autocorrelations of an AR(2) process are $r_1 = 0.5$ and $r_2 = 0.4$

A. What is the method of moments (Yule-Walker) estimate for ϕ_1?
B. What is the method of moments (Yule-Walker) estimate for ϕ_2?

Solution 12.1: See Cryer and Chan, top of page 150, equation 7.1.2:

$$\hat{\phi}_1 = \frac{r_1(1-r_2)}{1-r_1^2}$$

$$\hat{\phi}_2 = \frac{r_2 - r_1^2}{1-r_1^2}$$

Part A: The estimated ϕ_1 is (0.5 × (1 – 0.4)) / (1 – 0.5^2) = 0.4

Part B: The estimated ϕ_2 is (0.4 – 0.5^2)) / (1 – 0.5^2) = 0.2

Final exam problems may give ϕ_1 and ϕ_2 to derive ρ_1 and ρ_2; they may also give σ^2_ε and derive γ_0, γ_1, and γ_2; they may give r_1 and r_2 and derive ϕ_1 and ϕ_2; they may also give Var(Y) (γ_0) and derive σ^2_ε. The formulas are the same; compare them as you review the modules.

** Exercise 12.2: AR(2) process and method of moments practice problem

An AR(2) process with 100 observations has the following observed values:

$r_1 = 0.8$, $r_2 = 0.5$, $\bar{y} = 2$, and variance(Y) = 5.

A. What is the simple method of moments estimate of ϕ_1 used by Cryer and Chan?
B. What is the simple method of moments estimate of ϕ_2 used by Cryer and Chan?
C. What is the estimate of θ_0?
D. What is the estimate of σ^2_ε?

Solution 12.2: See Cryer and Chan, top of page 150, equation 7.1.2:

$$\hat{\phi}_1 = \frac{r_1(1-r_2)}{1-r_1^2}$$

$$\hat{\phi}_2 = \frac{r_2 - r_1^2}{1-r_1^2}$$

Part A: The estimated ϕ_1 is (0.8 × (1 – 0.5)) / (1 – 0.8^2) = 1.111

Part B: The estimated ϕ_2 is (0.5 – 0.8^2)) / (1 – 0.8^2) = -0.389

Part C: The estimated mean of the time series is 2. The mean = θ_0 / (1 – ϕ_1 – ϕ_2), so

$\theta_0 = \mu \times (1 - \phi_1 - \phi_2)$ = 2 × (1 – 1.111 – -0.389) = 0.556.

Part D: The estimate of σ^2_ε is (1 – 1.111 × 0.8 – (-0.389) × 0.5) × 5 = 1.529