TS Module 10 Sample autocorrelation functions practice problems

(The attached PDF file has better formatting.)

Question 1.1: Sample Autocorrelation Function

The sample autocorrelation of lag $k \approx (-0.366)^k$ for *all* $k > 1$, and the sample autocorrelation of lag 1 is –0.900. The time series is most likely which of the following choices?

A. AR(1)
B. MA(1)
C. ARMA(1,1)
D. ARIMA(1,1,1)
E. A random walk

Answer 1.1: C

A stationary autoregressive model has geometrically declining autocorrelations for lags more than its order. If the order is p, the lags for p+1 are higher are geometrically declining. This is true here, so we presume an AR(1) process.

If the time series is AR(1), the sample autocorrelation for lag 1 should be about –0.366. It is –0.900, so we assume the series also has a moving average component of order 1.

Question 1.2: Sample Autocorrelation Function

For a time series of 1,600 observations, the sample autocorrelation function of lag *k* is $\approx$ 0.366×1.2^{-k} for $k < 4$. For $k \geq 4$, the sample autocorrelations are normally distributed with a mean of zero and a standard deviation of 2.5%. The time series is probably

A. Stationary and Autoregressive of order 3
B. Stationary and Moving Average of order 3
C. Non-stationary
D. A random walk with a drift for three periods
E. A combination of stationary autoregressive of order 3 and a white noise process

Answer 1.2: B

Statement B: For $k \geq 4$ and 1,600 observations, the sample autocorrelations are normally distributed with a mean of zero and a standard deviation of 2.5%; these are the sample autocorrelations of a white noise process. A moving average time series has sample autocorrelations that drop off to a white noise process after its order (3 in this problem).

Statement A: An autoregressive process has geometrically declining autocorrelations for lags greater than its order.

Statements C and D: A non-stationary time series would not have autocorrelations that drop off to a random walk after 3 periods. A random walk is not stationary.

Statement E: A stochastic time series has white noise built in; adding white noise doesn't change anything.

Question 1.3: Sample Autocorrelation Function

If the sample autocorrelations for a time series of 1,600 observations for the first five lags are 0.461, 0.021, –0.017, 0.025, and –0.009, the time series is most likely which of the following choices?

A. AR(1) with $\varphi_1 \approx 0.45$
B. MA(1)
C. ARMA(1,1) with $\varphi_1 \approx 0.45$
D. ARIMA(1,1,1) with $\varphi_1 \approx 0.45$
E. A random walk

Answer 1.3: B

The sample autocorrelations decline to zero after the first lag, with random fluctuations within the white noise limits. The process is presumably moving average of order 1.

The process could also have an AR(1) parameter with $\varphi_1 < 0.15$, but we have no reason to assume an autoregressive parameter. If $\varphi_1 \approx 0.45$, the sample autocorrelation of lag 2 should be significantly more than zero.

Question 1.4: Covariances

We examine γ_0, γ_1, and γ_2, the covariances from a stationary time series for lags of 0, 1, and 2. Which of the following is true?

γ_0 is the variance, which is constant for a stationary time series, so the autocorrelations are the covariances divided by the variance. The autocorrelations have a maximum absolute value of one, and the variance is positive.

A. $\gamma_0 \geq \gamma_1$
B. $\gamma_1 \geq \gamma_2$
C. $\gamma_2 \geq \gamma_1$
D. $\gamma_1 + \gamma_2 \geq \gamma_0$
E. If $\gamma_1 \geq 0$, $\gamma_2 \geq 0$

Answer 1.4: A

The covariances of the time series can increase or decrease with the lag.

Illustration: For an MA(q) process with $\theta_j = 0$ for $1 \leq j \leq q-1$, the covariances are 0 for lags of 1 through q-1 but non-zero for a lag of q.

The variances of all elements of a stationary time series are the same, so none of the covariances can exceed the variance.

All five choices can be true. Only choice A is always true.

Question 1.5: Sample Autocorrelation Function

A time series is {8, 12, 11, 16, 10, 14, 9, 14, 13, 13, 9, 15}. What is $\hat{\rho}_2$,
the *sample* autocorrelation function at lag 2? Use the data in the table below.

T	Y_t	*(3)*	*(4)*	*(5)*
1	8	-4	4	16
2	12	0	0	0
3	11	-1	2	1
4	16	4	8	16
5	10	-2	6	4
6	14	2	4	4
7	9	-3	-3	9
8	14	2	2	4
9	13	1	-3	1
10	13	1	3	1
11	9	-3		9
12	15	3		9
Total	144	0.00	23.0	74.0

- Column 3 is $y_t - \bar{y}$
- Column 4 is $(y_t - \bar{y}) \times (y_{t+2}\ \bar{y})$
- Column 5 is $(y_t - \bar{y})^2$

A. –0.311
B. –0.120
C. +0.121
D. +0.311
E. +1.120

Answer 1.5: D

$\bar{y} = \sum y_t / 12 = 144/12 = 12.000$

$\hat{\rho}_2 = [\sum (y_t - \bar{y}) \times (y_{t+2} - \bar{y})] / \sum (y_t - \bar{y})^2 = 23 / 74 = 0.311$