TS Module 7 Stationary mixed processes

(The attached PDF file has better formatting.)

Time Series Practice Problems Stationarity

We model stationary time series by the techniques in the textbook. Most real world time series are not stationary, so take differences or logarithms to make them stationary.

Your student project make regress one time series on another or de-trend a time series to make it stationary. For example, a time series of average claim severity is not stationary.

- Average claim severity in real dollars (detrended) may be stationary.
- The residuals of a regression of average claim severity on inflation may be stationary.

*Question 1.2: Stationary

China’s economy is expanding rapidly. Inflation is 5.3% per annum and the number of cars is increasing 4.5% per annum. Chinese car makers are designing safer autos, and accident frequency is declining 20% per annum. Claim severity as a percentage of the auto’s value is not changing.

Which of the following time series is most likely to be stationary?

A. Annual auto insurance claim frequency
B. Annual auto insurance claim severity in deflated values.
C. Number of cars insured each year
D. Average car value each year
E. Total loss costs for all vehicles each year

Answer 1.2: B

A stationary process has a constant mean; it does not increase or decrease. In this scenario,

- The number of cars insured increases 4.5% per annum.
- The average car value and the claim severity increase 5.3% a year with inflation. In real (deflated) values, average claim severity may not be changing.
- Claim frequency decreases 10% a year with car safety.
- Total loss costs change by 1.045 × 1.053 / 1.200 – 1 = -8.30%

*Question 1.3: Autocorrelations

The absolute value of ρ_2 is always at least as great as the absolute value of ρ_{-3} for all but which of the following time series?

A. MA(1)
B. MA(2)
C. AR(1)
D. ARMA(1,1)
E. AR(3)

Answer 1.3: E

For MA(1) and MA(2), $\rho_3 = 0$.

For AR(1), AR(3), and ARMA(1,1), the value of ρ_k declines exponentially from the latest period with its own ARMA parameter. If the decline starts with Period 2, then $\rho_3 < \rho_2$. For a stationary process $\rho_3 = \rho_{-3}$.

*Question 1.4: Stationarity

Let $\Delta(y_t)$ be the first differences of the time $\{y_t\}$s .

If $\{y_t\}$ = 1.1 $y_{t-1} + \epsilon_t$,

which of the following time series is stationary?

$A.\ \{y_t\}$

$B.\ \{\ln(y_t)\}$

$C.\ \{\Delta(y_t)\}$

$D.\ \{\ln(\Delta(y_t))\}$

$E.\ \{\Delta(\ln(y_t))\}$

Answer 1.4: E

Choice A: This is an increasing autoregressive process with $\phi > 1$, which is not stationary.

Choice B: $ln(y_t) = ln(1.1) + ln(y_{t-1})$; this is a random walk with a drift of *ln*(1.1), which is not stationary.

Choice C: $\Delta(y_t) = 1.1\ y_{t-1} - y_{t-1} = 1.1\ y_{t-1} - 1.1\ y_{t-2} = 1.1 \times \Delta y_{t-1}$. This time series has the same pattern as the initial time series, though the values are smaller.

Choice D: Since the time series in Choice C has the same pattern as the initial time series, the time series in Choice D has the same pattern as the time series in Choice B.

Choice E: Choice B is a random walk, so Choice E is a white noise process, which is stationary.

*Question 1.5: Stationary Process

Which of the following is stationary?

A. An *autoregressive* process with $\varphi_1 = \varphi_2 = ½$.
B. An *autoregressive* process with $\varphi_1 = \varphi_2 = –1$.
C. An *autoregressive* process with $\varphi_1 = ½$, $\varphi_2 = ⅓$, $\varphi_3 = ¼$, $\varphi_4 = 1/5$, $\varphi_5 = 1/6$.
D. A *moving average* process with $\theta_1 = ½$, $\theta_2 = ⅓$, $\theta_3 = ¼$, $\theta_4 = 1/5$, $\theta_5 = 1/6$.
E. A *moving average* process of infinite order with $\theta_j = (–1)^j \times ½$: $\theta_1 = –½$, $\theta_2 = ½$, $\theta_3 = –1/2$, $\theta_4 = ½$, $\theta_5 = –½$, …

Answer 1.5: D

Jacob: Why is Statement A not stationary? Isn't this a simple average of the past two figures? Shouldn't this converge to a value between the most recent two figures?

Rachel: Consider the time series $Y_t = \delta + ½\, Y_{t-1} + ½\, Y_{t-2} + \epsilon$

- If $\delta > 0$, the forecast for period t as $t \to \infty$ is $+\infty$.
- If $\delta < 0$, the forecast for period t as $t \to \infty$ is $-\infty$.

This process is like a random walk with a drift.

Jacob: What if $\delta = 0$, so the process has no drift?

Rachel: We examine the variance of the forecasts. We consider first an autoregressive process with $\varphi_1 = 1$ and $\varphi_2 = 0$. Suppose the variance of the error term is σ^2.

- The variance of the one period ahead forecast is σ^2.
- The variance of the two periods ahead forecast is $\sigma^2 + \varphi_1^2$ × the variance of the one period ahead forecast = $2 \times \sigma^2$.
- The variance of the three periods ahead forecast is $\sigma^2 + \varphi_1^2$ × the variance of the two periods ahead forecast = $3 \times \sigma^2$.

The variance of the T periods ahead forecast is $\sigma^2 + \varphi_1^2$ × the variance of the T-1 periods ahead forecast = $T \times \sigma^2$. As $T \to \infty$, the variance $\to \infty$.

Jacob: What about the scenario where $\varphi_1 = \varphi_2 = ½$?

Rachel: The process has more pieces, but the reasoning is the same.

Another way to grasp the intuition is to repeat the reasoning above for $\varphi_1 = 0$ and $\varphi_2 = 1$.

The only change is one additional lag, so the variance of the T periods ahead forecast is $\sigma^2 + \varphi_1^2$ × the variance of the T-2 periods ahead forecast.

- If T is 1 or 2, the variance is σ^2.
- If T is 3 or 4, the variance is $2\sigma^2$.

In general, the variance is $\sigma^2 \times$ ceiling(½T).

The scenario where $\varphi_1 = \varphi_2 = ½$ is the average of the scenarios where

- the scenario where $\varphi_1 = 1$ and $\varphi_2 = 0$
- the scenario where $\varphi_1 = 0$ and $\varphi_2 = 1$