TS Module 5 Moving average MA(1) practice problems

(The attached PDF file has better formatting.)

**Exercise 5.1: MA(1) Process

An MA(1) process has $\theta_1 = 0.4$ and $\sigma^2_e = 4$.

A. What is the variance of Y_t?
B. What is γ_1 (covariance of Y_t with Y_{t-1})?
C. What is ρ_1 (correlation of Y_t with Y_{t-1})?

Part A: $Y_t = \mu + \epsilon_t - 0.4\ \epsilon_{t-1}$

- The error terms in different periods are independent.
- Y_t is the sum of two independent random variables with variances of 4 and $(-0.4)^2 \times 4$.

$$\text{Var}(Y_t) = \gamma_0 = 4 \times (1 + 0.4^2) = 4.640$$

(See Cryer and Chan, chapter 4, equation 4.2.2 at the bottom of page 57)

Part B: We compute the covariance of Y_t and Y_{t-1} = the covariance of

$$\mu + \epsilon_t - 0.4\ \epsilon_{t-1} \text{ and } \mu + \epsilon_{t-1} - 0.4\ \epsilon_{t-2}$$

The error terms are uncorrelated. The only non-zero covariance is from the ϵ_{t-1} term, which has a coefficient of $-\theta$ for Y_t and of +1 for Y_{t-1}. We multiply by the variance of ϵ.

$$\gamma_1 = \text{Covar}(Y_{t,t-1}) = -\ \theta_1\ \sigma^2_e = -0.4 \times 4 = -1.600$$

(See Cryer and Chan, chapter 4, equation 4.2.2 at the bottom of page 57)

Part C: The correlation is the covariance divided by the standard deviation of each term. The standard deviation is constant for all terms, so the product of two standard deviations is the variance of the error term.

$$\rho_1 = -\ \theta_1 / (1 + \theta_1^2) = -0.4 / (1 + 0.4^2) = -0.345$$

(See Cryer and Chan, chapter 4, equation 4.2.2 at the bottom of page 57)

The MA(1) process is simple. Final exam problems ask mostly AR(1), MA(1), AR(2), MA(2), and ARMA(1,1) among the stationary ARMA processes. Many exam problems invert the equations and derive the parameters from the observed sample autocorrelations.

**Question 5.2: Range of ρ_1 for an MA(1) Process

What is the range of ρ_1 for an MA(1) process?

A. $(-\infty, +\infty)$
B. $(-1, +1)$
C. $(-\frac{1}{2}, +\frac{1}{2})$
D. $(0, 1)$
E. $(0, +\infty)$

Answer 5.2: C

- The largest value possible for ρ_1 is ½ when $\theta_1 = -1$.
- The smallest value is $\rho_1 = -\frac{1}{2}$ when $\theta_1 = +1$.

(See Cryer and Chan page 58)

- $Y_t = \mu + \epsilon_t - \theta\, \epsilon_{t-1}$
- $Y_{t-1} = \mu + \epsilon_{t-1} - \theta\, \epsilon_{t-2}$

$\rho_1 = -\theta / (1 + \theta^2)$

To see the range of this expression, look at its reciprocal: $-(\theta + 1/\theta)$.

- As $\theta \to \pm 0$ or as $\theta \to \pm\infty$, this reciprocal $\to \pm\infty$.
- As $\theta \to \pm 1$, this reciprocal $\to \pm 2$.

*Question 5.3: MA(1) Process

A statistician estimates $\theta_1 = 0.4$ for an MA1 process from the value of ρ_1. What other value of θ_1 leads to the same ρ_1?

A. –0.4
B. 0.16
C. 0.6
D. 1.4
E. 2.5

Answer 5.3: E

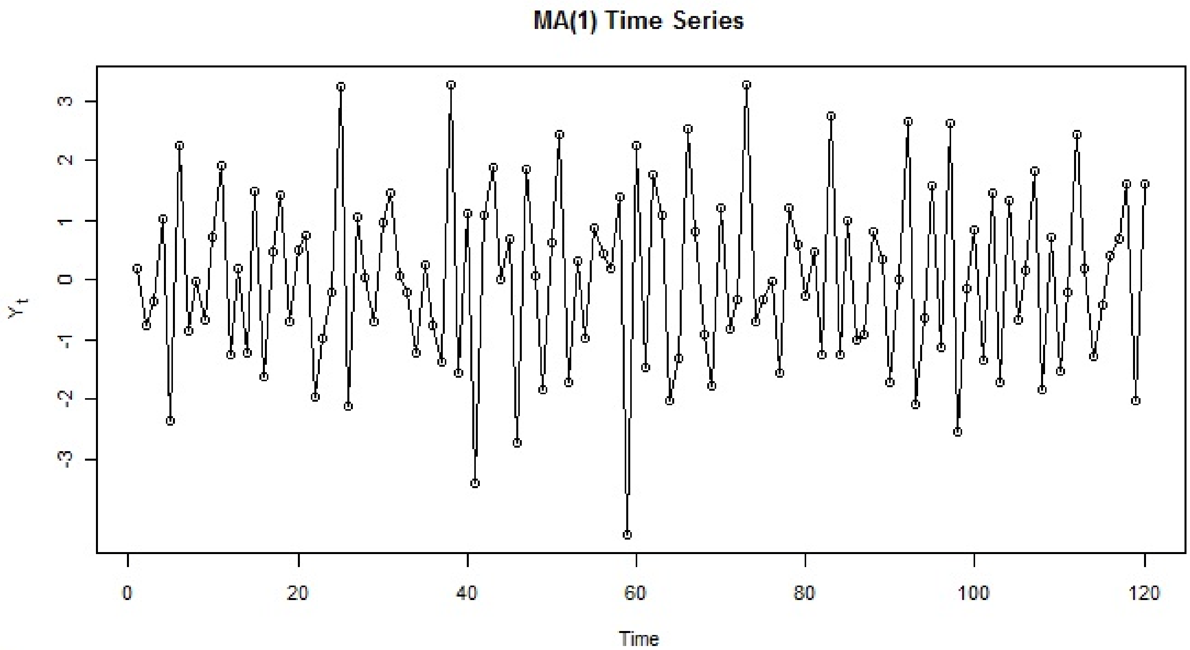
$\rho_1 = -\theta / (1 + \theta^2) \Rightarrow \theta$ and $1/\theta$ give the same ρ.

Note that if θ is negative, $1/\theta$ is also negative.

See Cryer and Chan, chapter 4, bottom of page 58. Final exam problems often ask for the invertable root, or the θ whose absolute value is less than one. An invertible MA(1) process has $-1 < \theta < +1$.

*Question 5.4: Time series graphs

The accompanying graph of a moving average time series is which of the following?



A. MA(1) with θ = +0.9
B. MA(1) with θ = -0.9
C. MA(1) with θ = +0.1
D. MA(1) with θ = -0.1
E. MA(1) with θ = 0

Answer 5.4: A

See Cryer and Chan, chapter 4, Exhibit 4.5 on page 61. The text on page 60 explains that

An MA(1) series with θ = +0.9 has $\rho_1 = -0.497$, giving moderately strong negative correlation at lag 1. In the graph, consecutive observations tend to be on opposite sides of the zero mean. If an observation is above the mean, the next observation tends to be below the mean, and vice versa. The plot has a jagged form.

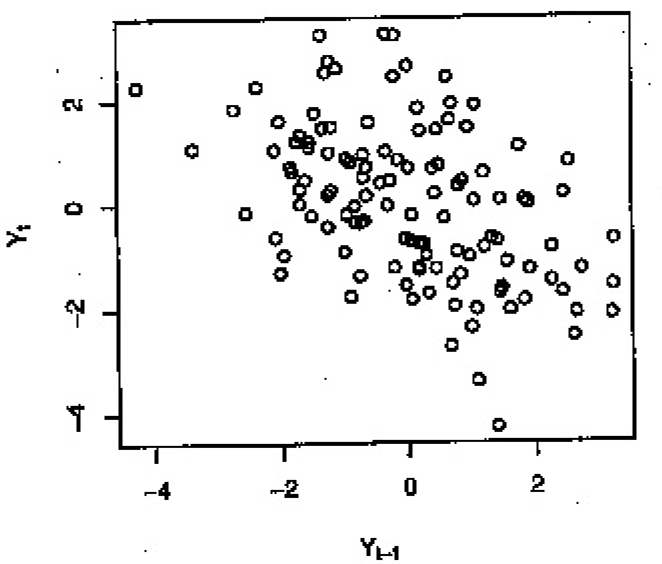
Final exam problems may give a plot of an MA(1) process with θ low or high and positive or negative.

Cryer and Chan generate this plot with the script:

```
win.graph(width = 7, height = 4, pointsize = 8)
plot(ma1.1.s, ylab=expression(Y[t]), yaxt="n", type = "o", las=1, main="MA(1) Time Series")
axis(side=2, at=c(-3:3))
```

*Question 5.5: Time series graphs

The accompanying graph of a moving average MA(1) time series has Y_{t-1} on the horizontal axis and Y_t on the vertical axis. Which of the following is the most likely value of θ?



A. MA(1) with θ = +0.9
B. MA(1) with θ = -0.9
C. MA(1) with θ = +0.1
D. MA(1) with θ = -0.1
E. MA(1) with θ = 0

Answer 5.5: A

The correlation of Y_{t-1} and Y_t is strongly *negative*, implying a *positive* θ close to one.

See Cryer and Chan, chapter 4, Exhibit 4.2 on page 59.

```
win.graph(width = 3, height = 3, pointsize = 8)
plot(y=ma1.1.s, x=zlag(ma1.1.s), ylab=expression(Y[t]), xlab=expression(Y[t-1]), type = "p", las=1, main="Plot of Y[t] vs Y[t-1] for MA(1) Time Series")
```