

TS Module 10 Autocorrelation functions

(The attached PDF file has better formatting.)

Time series practice problems partial autocorrelations

*Question 10.1: partial autocorrelations

A stationary ARMA process has $\rho_1 = 0.60$ and $\rho_2 = 0.52$. What is the partial autocorrelation of lag 2 (ϕ_{22})?

- A. 0.25
- B. 0.36
- C. 0.52
- D. 0.56
- E. 0.66

Answer 10.1: A

(Cryer and Chan, P113, equation 6.2.3)

$$\phi_{22} = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2}$$

$$(0.52 - 0.60^2) / (1 - 0.60^2) = 0.250$$

Jacob: What is the intuition for this formula?

Rachel: The partial autocorrelation of lag 2 says how much of the observed autocorrelation stems from ϕ_2 , after the effects of ϕ_1 are considered.

- The observed autocorrelation is ρ_2 .
- The effect of ϕ_1 is ρ_1^2 .
- The difference is attributed to ϕ_2 .

Jacob: What about the denominator of this formula?

Rachel: If ρ_1 is zero, the reasoning above is fine. Now suppose ϕ_1 is 80%, so this parameter gives ρ_2 of 64%. If the observed ρ_2 is greater, how much of the increase is caused by ϕ_2 ?

Illustration: If $\rho_2 = 76\%$, the extra correlation caused by ϕ_2 is $(76\% - 64\% = 12\%)$, which is one third of the remaining correlation $(1 - 64\% = 36\%)$.

*Question 10.2: partial autocorrelations

A stationary AR(1) process has $\phi = 50\%$.

What is $\phi_{11} - \phi_{22}$, the partial autocorrelation of lag 1 minus the partial autocorrelation of lag 2?

- A. -1.0
- B. -0.5
- C. 0
- D. +0.5
- E. +1.0

Answer 10.2: A

(P113, equation 6.2.3): For all AR(1) models:

- $\phi_{11} = 1$
- $\phi_{22} = 0$

*Question 10.3: Autocorrelations and partial autocorrelations

Which of the following are true?

1. For an AR(p) process, the autocorrelations decay exponentially as the lag increases for lags more than p .
2. For an MA(q) process, the autocorrelations decay exponentially as the lag increases for lags more than q .
3. For an AR(p) process, the partial autocorrelations decay exponentially as the lag increases for lags more than p .
4. For an MA(q) process, the partial autocorrelations decay exponentially as the lag increases for lags more than q .

- A. 1 and 2 only
- B. 3 and 4 only
- C. 1 and 3 only
- D. 2 and 4 only
- E. 1 and 4 only

Answer 10.3: E

We use sample autocorrelations and partial autocorrelations to select the type of model.

(Cryer and Chan, P113-114)