TS Module 7 Stationary mixed processes

(The attached PDF file has better formatting.)

Time series ARMA processes practice problems

*Question 7.1: ARMA(1,1)

An ARMA(1,1) process has $\rho_1 = 0$. Which of the following is true?

A. $\phi_1 = \theta_1$ or $\phi_1 \times \theta_1 = 1$
B. $\phi_1 = \theta_1$ and $\phi_1 \times \theta_1 = 1$
C. $\phi_1 = -\theta_1$ or $\phi_1 \times \theta_1 = -1$
D. $\phi_1 = -\theta_1$ and $\phi_1 \times \theta_1 = -1$
E. $\phi_1 = \theta_1$ and $\phi_1 \times \theta_1 = -1$

Answer 7.1: A

For an ARMA(1,1) process:

$$\rho_k = \frac{(1-\theta\phi)(\phi-\theta)}{1-2\theta\phi+\theta^2}\phi^{k-1} \qquad \text{for } k \geq 1$$

Intuition: If the residual in period t increases one unit:

- Moving average: θ_1 causes the forecast to decrease θ_1 units
- Autoregressive: ϕ_1 causes the forecast to increase ϕ_1 units

If $\phi_1 = \theta_1$, these two effects offset each other. A change in the period t value does not affect the period t+1 value, so $\rho_1 = 0$.

θ_1 and $1/\theta_1$ produce the same autocorrelation function.

$\phi_1 = \theta_1$ has the same effect as $\phi_1 = 1/\theta_1$ or $\phi_1 \times \theta_1 = 1$.

*Question 7.2: Exponential decay

For which of the following processes do the autocorrelations have exponential decay?

A. White noise
B. MA(1)
C. MA(2)
D. ARMA(1,1)
E. ARIMA(0,1,1)

Answer 7.2: D

See Cryer and Chan page 78: The ARMA(1,1) autocorrelation function decays exponentially as the lag k increases. The damping factor is ϕ, but the decay starts from initial value ρ_1, which also depends on θ. This is in contrast to AR(1) autocorrelation, which also decays with damping factor ϕ but always from initial value $\rho_o = 1$.

*Question 7.3: Exponential decay

The autocorrelation function of an ARMA(1,1) process has exponential decay with a damping factor of 0.4. Which of the following is true?

A. $\phi_1 - \theta_1 = 0.4$
B. $\phi_1 \times \theta_1 = 0.4$
C. $\phi_1 \times \theta_1 = 2.5$
D. $\phi_1 = 0.4$
E. $\theta_1 = -0.4$

Answer 7.3: D

*Question 7.4: Moving average and autoregressive processes

The moving average process $Y_t = e_t + 0.5\, e_{t-1} + 0.5^2\, e_{t-2} + 0.5^3\, e_{t-3} + \ldots$ is equivalent to what autoregressive process?

A. $Y_t = e_t + 0.5\, Y_{t-1} + 0.5^2\, Y_{t-2} + 0.5^3\, Y_{t-3} + \ldots$
B. $Y_t = e_t + 0.5\, Y_{t-1} + 0.5\, Y_{t-2} + 0.5\, Y_{t-3} + \ldots$
C. $Y_t = e_t + 0.5\, Y_{t-1}$
D. $Y_t = e_t - 0.5\, Y_{t-1} - 0.5^2\, Y_{t-2} - 0.5^3\, Y_{t-3} + \ldots$
E. $Y_t = e_t - 0.5\, Y_{t-1} + 0.5^2\, Y_{t-2} - 0.5^3\, Y_{t-3} + \ldots$

Answer 7.4: C

$$\begin{aligned} Y_t &= e_t + 0.5\, Y_{t-1} \\ &= e_t + 0.5\, (e_{t-1} + 0.5\, Y_{t-2}) \end{aligned}$$

Continue expanding to eliminate Y_t and remain with an infinite series of ϵ's.

See Cryer and Chan page 70, equation 4.3.8