TS Module 6 Moving average and autoregressive scatterplots
(The attached PDF file has better formatting.)
Cryer and Chan chapter 4 shows scatterplots for one and two lags for autoregressive and moving average processes. The final exam shows similar scatterplots and asks you to identify the time series process.

Exhibits 4.3 and 4.4 on pages 59 and 60 show scatterplots for an MA(1) process with $\theta=$ -0.9 , so $\rho_{1}$ is positive and $\rho_{2}$ is zero. The autocorrelations are evident from the scatter of points. Graphics iike those in the textbook are shown below.
\#\# MA(1) series with $\theta=-0.9$ of length $n=100$
ma1.n090 <- arima.sim(model $=\operatorname{list}(\mathrm{ma}=-\mathrm{c}(-0.90)), \mathrm{n}=100)$
par(mfrow=c(1,2))
plot(y = ma1.n090, $x=$ zzlag(vec $=$ ma1.n090, lag = 1), ylab $=$ expression(Y[t]), xlab $=$ expression (Y[t-1]), type = 'p')
title(main = expression(paste("Scatterplot of ",Y[t], " vs ",Y[t-1], " for MA(1) process with ", theta $==-0.9$, sep="") ), font $=2$, cex = 1.2)
$\operatorname{plot}(\mathrm{y}=\mathrm{ma1} . \mathrm{n} 090, \mathrm{x}=$ zzlag(vec $=$ ma1.n090, $\operatorname{lag}=2)$, ylab $=\operatorname{expression(Y[\mathrm {t}]),\text {xlab}=}$ expression (Y[t-2]), type='p')
title(main = expression(paste("Scatterplot of ",Y[t], " vs ",Y[t-2], " for MA(1) process with ", theta $==-0.9$, sep="") ), font = 2, cex = 1.2)
\#\# zzlag is like the zlag function in the TSA package

```
zzlag <- function (vec, lag =1) {
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len <- length(vec)
res <- vec[-((len-lag+1):len)]
res <- c(rep(NA, lag), res)
return(res)
\}


Scatterplot of $\mathrm{Y}_{\mathrm{t}}$ vs $\mathrm{Y}_{\mathrm{t}-2}$ for $\mathrm{MA}(1)$ process with $\theta=-0.9$


Exhibits 4.6 and 4.7 on pages 61 and 62 show scatterplots for an MA(1) process with $\theta=$ +0.9 , so $\rho_{1}$ is negative and $\rho_{2}$ is zero. The autocorrelations are evident from the scatter of points. Graphics iike those in the textbook are shown below.
\#\# MA(1) series with $\theta=0.9$ of length $n=100$
ma1.090 <- arima. $\operatorname{sim}($ model $=\operatorname{list}(m a=-c(0.90)), n=100)$
par(mfrow=c(1,2))
plot(y = ma1.090, $x=$ zzlag(vec = ma1.090, lag = 1), ylab = expression(Y[t]), xlab = expression (Y[t-1]), type = 'p')
title(main = expression(paste("Scatterplot of ",Y[t], " vs ",Y[t-1], " for MA(1) process with ", theta $==0.9$, sep="") ), font $=2$, cex $=1.2$ )
plot(y = ma1.090, $x=$ zzlag(vec = ma1.090, lag = 2), ylab = expression(Y[t]), xlab = expression (Y[t-2]), type='p')
title(main = expression(paste("Scatterplot of ",Y[t], " vs ",Y[t-2], " for MA(1) process with ", theta $==0.9$, sep="") $)$, font $=2$, cex $=1.2$ )

Scatterplot of $\mathrm{Y}_{\mathrm{t}}$ vs $\mathrm{Y}_{\mathrm{t}-1}$ for $\mathrm{MA}(1)$ process with $\theta=0.9$


Scatterplot of $\mathrm{Y}_{\mathrm{t}}$ vs $\mathrm{Y}_{\mathrm{t}-2}$ for $\mathrm{MA}(1)$ process with $\theta=0.9$


Exhibits 4.9 and 4.10 on page 64 show scatterplots for an MA(2) process. The values of $\theta_{1}$ and $\theta_{2}$ determine whether the autocorrelations are positive or negative. Final exam problems use parameter values that make the process clear.

Exhibits 4.14 and 4.15 on page 59 show scatterplots for an $\operatorname{AR}(1)$ process with $\theta=+0.9$, so $\rho_{1}$ and $\rho_{2}$ are both positive. Graphics iike those in the textbook are shown below.
\#\# AR(1) series with $\theta=0.9$ of length $n=100$
$\operatorname{ar1} .090<-\operatorname{arima} . \operatorname{sim}($ model $=\operatorname{list}(\operatorname{ar}=c(0.90)), n=100)$
par(mfrow=c(1,2))
plot(y = ar1.090, $x=$ zzlag(vec $=$ ar1.090, lag = 1), ylab $=$ expression(Y[t]), xlab = expression (Y[t-1]), type = 'p')
title(main = expression(paste("Scatterplot of ",Y[t], " vs ",Y[t-1], " for AR(1) process with ", phi $==0.9$, sep="") ), font $=2$, cex = 1.2)
$\operatorname{plot}(\mathrm{y}=\operatorname{ar1.090}, \mathrm{x}=\mathrm{zzlag}(\mathrm{vec}=\mathrm{ar} 1.090$, lag $=2)$, ylab $=\operatorname{expression(Y[\mathrm {t}]),\text {xlab}=}$ expression (Y[t-2]), type='p')
title(main = expression(paste("Scatterplot of ",Y[t], " vs ",Y[t-2], " for AR(1) process with ", phi $==0.9$, sep="") ), font $=2$, cex = 1.2)


Changing $\theta$ to -0.9 makes $\rho_{1}$ negative, but $\rho_{2}$ stays positive.
\#\# $\operatorname{AR}(1)$ series with $\theta=-0.9$ of length $n=100$
ar1.n090 <- arima.sim(model $=\operatorname{list}(\mathrm{ar}=\mathrm{c}(-0.90)), \mathrm{n}=100)$
$\operatorname{par}(m f r o w=c(1,2))$
plot( $\mathrm{y}=\operatorname{ar1} . \mathrm{n} 090, \mathrm{x}=\mathrm{zzlag}(\mathrm{vec}=\operatorname{ar} 1 . \mathrm{n} 090$, lag = 1), ylab $=\operatorname{expression(Y[t]),~xlab=}$ expression ( $\mathrm{Y}[\mathrm{t}-1 \mathrm{]}$ ), type = ' p ')
title(main = expression(paste("Scatterplot of ",Y[t], " vs ",Y[t-1], " for AR(1) process with ", phi $==-0.9$, sep="") ), font $=2$, cex = 1.2)
plot( $\mathrm{y}=\operatorname{ar1} . \mathrm{n} 090, \mathrm{x}=\mathrm{zzlag}(\mathrm{vec}=\operatorname{ar1} . \mathrm{n} 090, \operatorname{lag}=2$ ), ylab $=\operatorname{expression(Y[t]),~xlab=}$ expression ( $\mathrm{Y}[\mathrm{t}-2]$ ), type=' p ')
title(main = expression(paste("Scatterplot of ",Y[t], " vs ",Y[t-2], " for AR(1) process with ", phi $==-0.9$, sep="") ), font $=2$, cex $=1.2$ )

Scatterplot of $Y_{t}$ vs $Y_{t-1}$ for AR(1) process with $\phi=-0.9$


Scatterplot of $Y_{t}$ vs $Y_{t-2}$ for AR(1) process with $\phi=-0.9$


Final exam problems test the algebra and graphics of moving average and autoregressive processes: $\operatorname{MA}(1), \operatorname{MA}(2), \operatorname{AR}(1), \operatorname{AR}(2)$, and $\operatorname{ARMA}(1,1)$.

