TS Module 5 moving average MA(2) practice problems

(The attached PDF file has better formatting.)

**Exercise 5.1: Variance of moving average process

A moving average process of order 2 is $Y_t = e_t - 0.6\, e_{t-1} - e_{t-2}$, with $\sigma^2_e = 1$

A. What is γ_0, the variance of Y_t?
B. What is ρ_1, the autocorrelation of lag 1?
C. What is ρ_2, the autocorrelation of lag 2?

Part A: $\gamma_0 = (1 + \theta^2_1 + \theta^2_2) \times \sigma^2_\varepsilon = (1 + 0.6^2 + 1^2) \times 1 = 2.36$

Intuition: Y_t is the sum of three independent random variables, with variances of 1, 0.36, and 1.

See Cryer and Chan, chapter 4, page 62, equation at the bottom of the page.

Part B: See Cryer and Chan, chapter 4, page 63, equation 4.2.3.

$$\rho_1 = \frac{-\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2} \qquad \rho_1 = \frac{-\theta_1+\theta_1\theta_2}{1+\theta_1^2+\theta_2^2}$$

$(-0.6 + 0.6 \times 1) / (1 + 0.6^2 + 1^2) = 0/2.36$

ρ_1 is the correlation of $e_t - 0.6\, e_{t-1} - e_{t-2}$ with $e_{t-1} - 0.6\, e_{t-2} - e_{t-3}$

- The numerator has two non-zero terms: $-0.6\, e^2_{t-1}$ and $+0.6\, e^2_{t-2}$
- The error terms have the same variance, so these two terms cancel out.

Jacob: Does $\rho_2 = 0$ mean that the autocorrelation of lag 2 in this time series is zero?

Rachel: Actual time series are finite, so the autocorrelations have random fluctuations. If this time series has 100 observations, the autocorrelation ρ_2 is distributed as a normal random variable with a mean of zero and a standard error of $1/\sqrt{100} = 0.10$. The observed autocorrelation is not zero. But if the time series is infinite, the autocorrelation of lag 2 approaches zero.

Jacob: How should we think of this? Can one see the correlation intuitively?

Rachel: e_t is correlated with e_{t-1} two ways:

- Directly, with a correlation of $-\theta_1$.
- Indirectly through e_{t-2} and back to e_{t-1}, with a correlation of $-\theta_2 \times -\theta_1$.

Moving from e_{t-1} to e_{t-2} is like moving from e_{t-2} to e_{t-1}. We deal with these relations in more detail in modules 19 and 20 (seasonal time series), where $\rho_{11} = \rho_{13}$.

Part C: See Cryer and Chan, chapter 4, page 63, equation 4.2.3

$$\rho_2 = \frac{-\theta_2}{1+\theta_1^2+\theta_2^2}$$

$(-1) / (1 + 0.6^2 + 1^2) = -1/2.36 = -0.424$

Intuition: ρ_2 is the correlation of $e_t - 0.6\, e_{t-1} - e_{t-2}$ with $e_{t-2} - 0.6\, e_{t-3} - e_{t-4}$

- The numerator has one non-zero terms: $-1 \times e^2_{t-2}$
- The denominator is the variance γ_0