TS Module 2 Time series concepts practice problems

(The attached PDF file has better formatting.)

Time series practice problems variances and covariances

*Question 2.1: Random walk

The time series $Y_t = Y_{t-1} + e_t$ is a random walk with $\sigma^2_e = 0.25$ and $Y_t = 0$ for $t < 1$.

What is the standard deviation of Y_9?

A. 0.15
B. 0.25
C. 0.50
D. 1.50
E. 2.25

Answer 2.1: D

- $Y_t = \epsilon_1 + \epsilon_2 + \ldots + \epsilon_9$
- The error terms are independent, with a variance of 0.25 each.
- The variance of Y_9 is $9 \times 0.25 = 2.25$.
- The standard deviation of Y_9 is $(9 \times 0.25)^{0.5} = 1.50$.

*Question 2.2: Equally weighted moving average

Let $Y_t = ½ \times (\epsilon_t + \epsilon_{t-1})$. What is $\rho_{t,t-1}$, the correlation of Y_t and Y_{t-1}?

A. –1
B. –0.5
C. 0
D. 0.5
E. 1

Answer 2.2: D

Let σ^2 = the variance of the error term.

- The covariance of Y_t with Y_{t-1} = covariance $(½ \times (\epsilon_t + \epsilon_{t-1}), ½ \times (\epsilon_{t-1} + \epsilon_{t-2})) = ¼ \sigma^2$.
- The variance of Y_t is $¼ \times 2 \sigma^2 = ½ \sigma^2$.
- The correlation of Y_t with $Y_{t-1} = ¼ \sigma^2 / ½ \sigma^2 = 0.500$.

See Cryer and Chan page 15, equation 2.2.16:

$\rho_{t,s} = 0.5$ for |t–s| = 1

(2.2.16)

*Question 2.3: Random walk

- Let Y_t be a random walk with zero drift: $Y_t = Y_{t-1} + \epsilon_t$, with $\sigma^2_e = 1$.
- The time series starts at t = 1. (This time series is not stationary.)

What is $\rho_{4,25}$?

A. 0.00
B. 0.16
C. 0.40
D. 4.00
E. 25.0

Answer 2.3: C

$(4/25)^{0.5} = 0.400$

(See Cryer and Chan equation 2.2.13)

*Question 2.4: Stationary time series

- X_t is a stationary time series.
- $Y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + X_t$

Which of the following is a stationary time series?

A. $Y_t - 2 Y_{t-1} - Y_{t-2}$
B. $Y_t - 2 Y_{t-1} + Y_{t-2}$
C. $Y_t + 2 Y_{t-1} + Y_{t-2}$
D. $Y_t + 2 Y_{t-1} - Y_{t-2}$
E. All of A, B, C, and D are stationary

Answer 2.4: B

See Cryer and Chan page 20, exercise 2.9.

Choice B is the second difference. Y_t is a quadratic function of t, so the second difference is a constant. The second difference of X is stationary.