

TS Module 12: Parameter estimation method of moments practice problems

(The attached PDF file has better formatting.)

Time series practice problems on Yule-Walker equations

We use the Yule-Walker equations to derive initial estimates of the ARMA coefficients. Know how to solve the Yule-Walker equations for AR(1), AR(2), and MA(1) processes.

- A student project might use the Yule-Walker equations for MA(1) and MA(2) models.
- For the final exam, focus on the equations for AR(1), AR(2), and MA(1) models.

The Cryer and Chan textbook shows the efficiency of the Yule-Walker estimates for various processes. Know when these equations are good estimators and when they are not. (They are not efficient for moving average and mixed models.)

Many exam problems specify that the time series parameters lie between -1 and $+1$. The term used in the textbook is that the time series is invertible.

*Exercise 12.1: MA(1) model and Yule-Walker equations

An MA(1) model has an estimated ρ_1 of -0.35 . What is the Yule-Walker initial estimate for θ_1 if it lies between -1 and $+1$?

Solution 12.1: An MA(1) model has $\rho_1 = \frac{-\theta_1}{(1+\theta_1^2)}$

We invert the equation to get $\theta_1 = \frac{-1 \pm \sqrt{1-4\rho_1^2}}{2\rho_1}$

We compute $(-1 + (1 - 4 \times 0.35^2)^{0.5}) / (2 \times -0.35) = 0.408$

Note: For an MA(1) process, Cryer and Chan use θ instead of θ_1 .

For a given value of ρ_1 , two values of θ_1 may solve the Yule-Walker equation. The exam problem may specify bounds for θ_1 , such as an absolute value less than one. The textbook expresses this as the MA(1) model is invertible.

For an invertible MA(1) model, ρ_1 and θ_1 has opposite signs. This reflects the sign convention for the moving average parameter.

Know several limiting cases.

- As $\theta_1 \rightarrow$ zero, $\rho_1 \rightarrow$ zero
- As $\theta_1 \rightarrow$ one, $\rho_1 \rightarrow$ negative one half (-0.5)
- As $\theta_1 \rightarrow$ infinity, $\rho_1 \rightarrow$ negative one (-1)

Reversing the sign of θ_1 reverses the sign of ρ_1 .

Jacob: If the Yule-Walker equations are not efficient for moving average models, why do we use them?

Rachel: We use them for initial estimates, not final estimates. Autoregressive processes can be estimated by linear regression. We need nonlinear regression for moving average processes, which can not be solved by hand (or by Excel without extra macros or add-ins.)