TS Module 7 Oscillating models and mean reversion practice problems

(The attached PDF file has better formatting.)

Time series practice problems oscillating models and mean reversion

Most exam problems use simple ARMA processes with both p and q equal to or less than 2. Know the pattern of the forecasts: geometric decline for AR(1) with $\phi > 0$, oscillating for AR(1) with $\phi < 0$, a drop for moving average processes, and a combination for ARMA processes.

Moving average terms may cause the forecasts to move away from the mean for one or two periods in some scenarios.

Illustration: Assume the mean of the time series is 0; $\varphi_1 = 0.9$, so mean reversion is weak; and $\theta_1 = -2.0$, so the moving average effect is strong. In Period T, the value is 0 and the residual is 0. In Period 1, suppose the actual value is 1.0, so the residual is 1.0.

- The forecast for Period 2 is $0.9 \times 1.0 + 1.0 \times (-(-2.0)) = 2.9$.
- Higher order forecasts have mean reversion of 10% towards 0. The forecast for Period 3 is $2.9 \times 90\% = 2.610$.

Know the rules for each type of ARIMA process.

- The forecasts of a moving average MA(q) model can move away from the mean of the time series *at most* for at most q periods.
- The forecasts of a stationary autoregressive AR(p) model are mean reverting from the weighted average of the most recent p values.

Illustration: An AR(2) process with $\mu = 0$, $\phi_1 = 0.1$, and $\phi_2 = 0.8$, might have values for the first five periods of 0, 0, 0, 1, 0. The forecasts of the next values are 0.8, 0.08, 0.648, and so forth. The mean reversion exists, but it may not be clear at first.

- Know which ARIMA models oscillate about their means, and which models approach the mean asymptotically.
- Know when a model reverts toward the mean in the next period and when it might drift away from the mean for one or two periods. Mean reversion and drifts refer to the forecasts. An ARIMA process is stochastic, so the actual values may be farther away from the mean for any process.

*Question 7.1: Oscillating Models

All but which of the following models oscillate about their means?

A. AR(1) with $\mu = 0$ and $\varphi_1 = –0.667$.
B. AR(1) with $\mu = 10$ and $\varphi_1 = –0.333$.
C. ARMA(1,1) with $\mu = 0$, $\varphi_1 = –0.667$, and $\theta_1 = +0.333$
D. ARMA(1,1) with $\mu = 0$, $\varphi_1 = +0.667$, and $\theta_1 = –0.333$
E. ARMA(1,1) with $\mu = 0$, $\varphi_1 = –0.667$, and $\theta_1 = –0.333$

Answer 7.1: D

An ARMA(1,1) process with a *negative* φ_1 parameter oscillates about its mean. The mean itself is not relevant and the θ_1 parameter is not relevant.

A stationary process has a mean. As the lag grows, the forecast approaches the mean.

In the short run, an ARMA process may not be mean reverting. For any moving average process and for an autoregressive process of order $p > 1$, the one period ahead forecast may be farther from the mean than the most recent value of the time series.

The exam problem may give the one or two most recent values of the time series and the coming one or two forecasts. It may ask which ARMA process is most likely or least likely.

- To solve the problem, first see if a process is possible.
- Among the possible models, use the cdf of the normal distribution to see which models are most likely.

The exam problems use simple scenarios for which you don't need a cdf table.

Jacob: Is the mean for the current term or the forecast with an unbounded lag? Suppose interest rates follow a monthly AR(1) process, with μ = 5%. If the current interest rate is 4% and ϕ = 95%, next month's interest rate is 4.05%, not 5%. Why do we call 5% the mean?

Rachel: If we don't know the interest rate now, the best estimate of next month's rate is 5%.

*Question 7.2: Non-Mean Reverting Models

An ARMA process of 50 values has μ = 100, σ^2 = 4, and y_{50} = 104. The forecast for y_{51} is 105.

The ARMA process is most likely which of the following?

A. AR(1) with φ_1 = –0.750.
B. AR(1) with φ_1 = –0.250.
C. ARMA(1,1) with φ_1 = –0.750, and θ_1 = +0.250
D. ARMA(1,1) with φ_1 = +0.750, and θ_1 = –2.500
E. ARMA(1,1) with φ_1 = –0.750, and θ_1 = –0.250

Answer 7.2: D

The model can not be A or B, since

- For Choice A, the one period ahead forecast is 97.
- For Choice B, the one period ahead forecast is 99.

For Choice C, a one period ahead forecast of 105 is eight units above 97.
θ_1 is +0.250, so the residual for Period 50 must be –32.
With σ^2 = 4, σ = 2, and a residual of –32 is 16 standard deviations.

For Choice E, a one period ahead forecast of 105 is eight units above 97.
θ_1 is –0.250, so the residual for Period 50 must be +32.
With σ^2 = 4, σ = 2, and a residual of +32 is –16 standard deviations.

For Choice D, a one period ahead forecast of 105 is two units above 103.
θ_1 is –2.500, so the residual for Period 50 must be +0.800.
With σ^2 = 4, σ = 2, and a residual of 0.800 is 0.4 standard deviations.

The probability of 16 standard deviations from the mean of zero is almost nil, so Choices C and E are not correct.

The probability of 0.4 (or more) standard deviations from the mean of zero is high, so Choice D is correct.