The first differences are an AR(1) model: $\Delta y_t = 5 + \varphi_1 \Delta y_{t-1} + \varepsilon_t$.

- *Step 1:* Determine the most recent value of the autoregressive model from the most recent two values of the original time series: $\Delta y_{40} = y_{40} - y_{39} = 60 - 50 = 10$
- *Step 2:* Convert forecasts of the time series to forecasts of the first differences for the one period ahead forecast. The forecast for period 41 is 60, so the forecasted first difference is 60 – 60 = 0.
- *Step 3:* Find the parameter ϕ_1 from the 1 period ahead forecast. $\Delta y_{41} = 5 + \varphi_1 \times 10 = 0 \Rrightarrow \varphi_1 = -0.5$. $\theta_0 = 5$ is not the mean of the AR(1) process; it is the constant term. The AR(1) process can be written two ways: $Y_t = \theta_0 + \phi_1 \times Y_{t-1}$ or $Y_t - \mu = \phi_1 \times (Y_{t-1} - \mu)$. The textbook uses both formats, but θ_0 and μ are different values.
- *Step 4:* Solve for the two period ahead forecast from the autoregressive equation. $\Delta y_{42} = 5 - 0.5 \times 0 = 5$
- *Step 5:* Convert the forecast of the first differences for two periods to the initial time series. $y_{42} = y_{41} + 5 = 65$

The textbook has formulas for forecasts and variances of ARIMA processes.

- If you understand the intuition, the formulas are easy to recall and provide a good check on your work.
- If you do not understand the intuition, you will mix up the formulas for the various ARIMA processes. Focus on the intuition. After a few problems, it is easy.