Corpfin module 8: Risk and the cost of capital

(The attached PDF file has better formatting.)

Brealey and Myers, Chapter 9, Risk and the cost of capital

** Exercise 8.1: WACC

The risk-free rate is 10% per annum, the market risk premium is 8%, and the tax rate is 35%.

- A firm's equity has a CAPM β of 50%.
- The firm's debt yields 10% per annum.
- The debt-to-value ratio is 40%. is 11%.

A. What is the cost of equity capital?
B. What is the after-tax weighted average cost of capital?
C. What is the debt-to-equity ratio?

Part A: The cost of equity capital is the risk-free rate + the CAPM β × the market risk premium:

$$10\% + 50\% \times 8\% = 14\%.$$

Part B: The WACC = $D/V \times (1 - T_c) \times r_D + E/V \times r_E$, where

- Tc is the tax rate
- r_D is the cost of debt capital
- r_E is the cost of equity capital
- D is the market value of debt
- E is the market value of equity
- V = D + E

E/V = 1 – D/V, so WACC = 40% × (1 – 35%) × 10% + (1 – 40%) × 14% = 11%.

Part C: The debt-to-equity ratio D/E = D/V ÷ (1 – D/V) = 40% / (1 – 40%) = 66.67%.

Note: A final exam problem may give the debt-to-equity ratio, from which you determine the debt to value ratio.

Some final exam problems give the weighted average cost of capital, from which you derive the cost of equity capital, the cost of debt capital, the debt-to-equity ratio, or the debt to value ratio. The following exercise shows a sample computation.

** Exercise 8.2: WACC

The risk-free rate is 10% per annum, the market risk premium is 8%, and the tax rate is 35%.

- A firm's equity has a CAPM β of 50%.
- The firm's debt yields 10% per annum.
- The after-tax weighted average cost of capital is 11%.

A. What is the cost of equity capital?
B. What is the debt-to-value ratio?
C. What is the debt-to-equity ratio?

Part A: The cost of equity capital is the risk-free rate + the CAPM β × the market risk premium:

$$10\% + 50\% \times 8\% = 14\%.$$

Part B: The WACC = $D/V \times (1 - T_c) \times r_D + E/V \times r_E$, where

- Tc is the tax rate
- r_D is the cost of debt capital
- r_E is the cost of equity capital
- D is the market value of debt
- E is the market value of equity
- V = D + E

$E/V = 1 - D/V$, so $D/V = (WACC - r_E) / [(1 - T_c) \times r_D - r_E) = (11\% - 14\%) / ((1 - 35\%) \times 10\% - 14\%) = 40\%$.

Part C: The debt-to-equity ratio $D/E = D/V \div (1 - D/V) = 40\% / (1 - 40\%) = 66.67\%$.

** Exercise 8.3: Asset betas

Which of each pair of projects below is likely to have the higher asset beta?

A. The sales force for Project Y is paid a fixed annual salary; Project Z's sales force is paid by commission.
B. Project Y runs a first-class airline in Dubai; Project Z sells breakfast cereals.

Part A: The fixed annual salary is cost in good and bad years. During recessions, Project Y may have costs it can not pay. Commissions depend on sales; during recessions, costs for Project Z decline. Project Y has more systematic risk, since its losses occur in recessions, so it has the higher beta.

Part B: A first class airline does well when the overall economy improves; it does poorly when the economy is poor. It is correlated with overall market results, so it has a higher asset beta. Breakfast cereals sell in all years (people eat breakfast in good and bad economic times), so it has a low asset beta.

** Exercise 8.4: Certainty equivalent cash flows

A project has projected cash flows of $110 at time t=1 and $121 at time t=2.

The risk-free interest rate is 5% per annum, the market risk premium is 10%, and the project has a CAPM beta of 50%.

C. What is the opportunity cost of capital for this project?
D. What is the present value of the project at time t=0?
E. What are the certainty equivalent cash flows at time t=1 and time t=2?
F. What are the ratios of the certainty equivalent cash flows to the expected cash flows at times t=1 and t=2?

Part A: The opportunity cost of capital for this project is 5% + 50% × 10% = 10%.

Part B: The present value of the project at time t=0 is $\$110 / 1.1^1 + \$121 / 1.1^2 = \$200$.

Part C: The certainty equivalent cash flows are discounted at 5%, not 10%.

- Time t=1: $Z / 1.05^1 = \$110 / 1.10^1 \Rightarrow Z = 1.05^1 \times \$110 / 1.10^1 = \$105.00$.
- Time t=2: $Z / 1.05^2 = \$121 / 1.10^2 \Rightarrow Z = 1.05^2 \times \$121 / 1.10^2 = \$110.25$.

Part C: The ratios of the certainty equivalent cash flows to the expected cash flows at times t=1 and t=2 are

- Time t=1: $105.00 / $110 = 0.9545.
- Time t=2: $110.25 / $121 = 0.9112.

** Exercise 8.5: Certainty equivalent cash flows

A project has the same projected cash flows at times t=1 and t=2.

The risk-free interest rate is 4% per annum, the market risk premium is 8%, and the project's beta is 75%.

A. What is the opportunity cost of capital for this project?
B. What are the certainty equivalent cash flows at time t=1 and time t=2?
C. What is the ratio of the certainty equivalent cash flow at time t=1 to the certainty equivalent cash flow at time t=2?

Part A: The opportunity cost of capital for this project is 4% + 75% × 8% = 10%.

Part B: The certainty equivalent cash flows are discounted at 4%, not 10%. If the expected cash flows are Y, the certainty equivalent cash flows Z are

- Time t=1: $Z / 1.04^1 = Y / 1.10^1 \Rightarrow Z = 1.04^1 \times Y / 1.10^1$.
- Time t=2: $Z / 1.04^2 = Y / 1.10^2 \Rightarrow Z = 1.04^2 \times Y / 1.10^2$.

Part C: The ratio of the certainty equivalent cash flow at time t=1 to the certainty equivalent cash flow at time t=2 is $[1.04^1 \times Y / 1.10^1] \div [1.04^2 \times Y / 1.10^2] = 1.10 / 1.04 = 1.0577$.

** Exercise 8.6: Systematic vs unique risk

Firm ABC's stock returns has a standard deviation of 25%. A regression of the firm's returns on the overall market returns has a beta of 0.82 (with a standard error of 0.18) and an R^2 of 0.25.

The risk-free rate is 5% and the expected market return is 12%.

A. What proportion of the stock's risk is market risk and what proportion is specific risk?
B. What is the variance of the firm's stock returns?
C. What is the market variance?
D. What is the specific variance?
E. What is the 95% confidence interval for the firm's beta? (The *z*-value for a 95% confidence interval is 1.960.)
F. If the CAPM is correct, what is the expected return of the firm's stock?
G. If the market return is zero this year, what is the expected return of the firm's stock?

Part A: The R^2 is 25%, implying that overall market movements account for 25% of the firm's risk: 25% is market risk and 1 – 25% = 75% is specific risk.

Part B: The variance is the square of the standard deviation: $25\%^2 = 0.25^2 = 0.06250 = 6.25\%$.

Part C: The market variance is 25% × 6.25% = 0.015625 = 1.5625%

Part D: The specific variance is 75% × 6.25% = 0.046875 = 4.6875%

Part E: The 95% confidence interval using a z-value of 1.960 is 0.82 ± 1.960 × 0.18 = (0.4672, 1.1728).

Part F: The expected return on the firm's stock is 5% + 0.82 × (12% – 5%) = 10.74%.

Part G: If the overall market return one year is zero, the expected return on the firm's stock is 5% + 0.82 × (0% – 5%) = 0.90%.