Corporate Finance Mod 23: Options, effects on prices, practice problems

** Exercise 23.1: Effects on option prices

- A stock trades at S (the current stock price).
- The stock price volatility is σ.
- The risk-free rate is *r*% *per annum*.
- European call and put options trade with strike prices of X (exercise price).

A. If S increases, do the option prices increase or decrease?
B. If X increases, do the option prices increase or decrease?
C. If σ increases, do the option prices increase or decrease?
D. If *r* increases, do the option prices increase or decrease?
E. What happens if the firm pays a larger dividend than expected?

Part A: As the stock price increases, the call price increases and the put price decreases.

The payoff of a call option is the stock price minus the exercise price, with a minimum of zero: max(0, S – X). As S increases, the payoff increases or stays the same.

The stock price at the expiration date is stochastic. A higher stock price now means a likely higher stock price at expiration. A higher stock price at inception always causes a higher value for the call option.

The payoff of a put option is the exercise price minus the stock price, with a minimum of zero: max(0, X – S). As S increases, the payoff decreases or stays the same.

The stock price at the expiration date is stochastic. A higher stock price now means a likely higher stock price at expiration. A higher stock price at inception always causes a lower value for the put option.

Part B: As the exercise price increases, the call price decreases and the put price increases. The logic is the same as for Part A.

Part C: Higher volatility of the stock price raises the value of both call and put options. We show this by intuition and by the Black-Scholes formula.

Intuition: The option value rises if the stock price moves favorably: up for a call option and down for a put option. The decrease in value is bounded at zero if the stock price moves adversely.

Suppose the stock price is $100, the strike price (exercise price) is $100, the risk-free interest rate is zero, and the option has one year to maturity. (The result is true for any stock price and exercise price; the figrures make the intuition easier to grasp). The stock price moves up by Z% or down by Z% over the year with equal risk-neutral probabilities. Higher volatility means a higher value of Z.

If Z% = 1%, the value of the call option at expiration is $1 if the stock price moves up and $0 if the stock price moves down. The expected value of the call option at expiration is $0.50. The risk-free interest rate is zero, so this is also the present value of the call option.

If Z% = 10%, the value of the call option at expiration is $10 if the stock price moves up and $0 if the stock price moves down. The expected value of the call option at expiration is $5.00. The risk-free interest rate is zero, so this is also the present value of the call option.

For any Z, the value of the call option at expiration is $Z if the stock price moves up and $0 if the stock price moves down. The expected value of the call option at expiration is ½ × $Z. The risk-free interest rate is zero, so this is also the present value of the call option.

For the put option, we inter-change “up” and “down,” with the same result.

The Black-Scholes formula gives the same result. The Black-Scholes formula has four elements: S, PV(X), d_1, and d_2. As the volatility σ increases, d_1 increases and d_2 decreases.

For a call option, $N(d_1)$ increases and $N(d_2)$ decreases. $S \times N(d_1)$ increases and $PV(X) \times N(d_2)$ decreases, so the value of the call option increases.

For a put option, $N(-d_1)$ decreases and $N(-d_2)$ increases. $S \times N(-d_1)$ decreases and $PV(X) \times N(-d_2)$ increases, so the value of the put option increases.

Part D: A higher risk-free rate increases the value of the call option and decreases the value of the put option.

Intuition: Suppose the stock price is $80, the exercise price of a call option is $80, and the option has three months to maturity. The option has two benefits for the investor:

- *Volatility value:* An investor who buys the stock in the free market pays $80. With the option, the investor can wait three months: if the stock price increases, he buys the stock for $80, and if the stock price decreases, he can buy the stock in the free market for less.
- *Interest value:* The investor gains from the option even if the stock price remains $80. The investor can invest the cash at the risk-free rate for three months before paying for the stock.

An increase in the risk-free rate that is not accompanied by an increase in the stock’s return or its volatility does not affect the volatility value of the option. But it increases the interest value by the additional interest income over the term of the call option.

A put option has the same volatility value as the call option. But the interest value for a put option is negative. If the stock price remains $80 over the term of the put option, an investor who sells the stock immediately in the free market can invest the exercise price for the term of the option. An investor who sells the stock by exercising the put option doesn’t receive the cash until the expiration date.

Part E: The effects of dividends depend on their influence on the stock price.

Sometimes a large dividend indicates that the firm is doing better than it expected. If investors are persuaded by the large dividend that the firm is doing unusually well, the stock price might rise, which raises the value of the call option and reduces the value of the put option.

The final exam problems generally say that the larger or smaller dividend does not convey any information to investors. A larger dividend simply means the firm is paying out its cash to shareholders, so the stock price falls, the value of the call option, and the value of the put option rises.