Corporate Finance Mod20: Option Combinations, practice problems

** Exercise 20.1: Put and call options

An investor buys two three month European options on stock ABC:

- A call option with an exercise price of 60.
- A put option with an exercise price of 70.

The investor also sells two three month European options on stock ABC:

- A call option with an exercise price of 80.
- A put option with an exercise price of 50.

A. At what ending stock price is the payoff to the investor greatest?
B. At what ending stock price is the payoff to the investor smallest?

Parts A and B: We compute the net payoff as a function of the ending stock price S.

If the ending stock price is less than 50, both call options expire worthless and both put options are exercised. The investor gets 70 – S and pays 50 – S, for a net payoff of (70 – S) – (50 – S) = 20.

If the ending stock price is more than 80, both put options are exercised and the call options expire worthless. The investor gets S – 50 and pays S – 70, for a net payoff of (S – 60) – (S – 80) = 20.

If the ending stock price is between 50 and 60, only the put option at 70 is exercised. The investor gets 70 – S, which is between 10 and 20.

If the ending stock price is between 60 and 70, the put option at 70 and the call option at 60 are exercised. The investor gets (70 – S) + (S – 60), which is 10.

If the ending stock price is between 70 and 80, only the call option at 60 is exercised. The investor gets (S – 60), which is between 10 and 20.

If the ending stock price is more than 80, both put options are exercised and the call options expire worthless. The investor gets S – 50 and pays S – 70, for a net payoff of (S – 60) – (S – 80) = 20.

In sum: the highest payoff is for an ending stock price less than 50 or more than 80; the lowest payoff is for an ending stock price between 60 and 70.

** Exercise 20.2: Option Combinations

An investor enters into the following transactions for six month options on Stock ABC:

- Sells one put option at an exercise price of $70.
- Sells one put option at an exercise price of $90.
- Buys **two** put options at an exercise price of $80.

What is the payoff to the investor if the ending stock price is

A. Less than 70
B. Between 70 and 80
C. Between 80 and 90
D. More than 90

Part A: If the stock price is $70 or less, the short put options are worth $70 – S and $90 – S. Together, they are worth $160 – 2S. The investor who sells the put options must pay this amount. The two long put options are worth 2 × ($80 – S) = $160 – 2S. This offsets the loss to the investor from the sale of the two put options at $70 and $90. The net payoff is zero.

Part D: If the stock price is $90 or more, all the put options expire worthless, and the net payoff to the investor is zero.

Part B: If the stock price is between $70 and $80, the put option at $70 expires worthless, the two put options at $80 are worth 2 × ($80 – S), and the put option at $90 is worth $90 – S. The net payoff to the investor is 2 × ($80 – S) – ($90 – S) = $70 – S. S is between $70 and $80, so the net payoff is negative, with a minimum of –$10 at S = $80.

Part C: If the stock price is between $80 and $90, the put option at $70 and $80 expire worthless, and the put option at $90 is worth $90 – S. The net payoff to the investor is –($90 – S). S is between $80 and $90, so the net payoff is negative, with a minimum of –$10 at S = $80.

- The maximum value of a *long* butterfly is at the midpoint.
- This is a *short* butterfly, so its minimum value is at the midpoint.

** Exercise 20.3: Options Payoffs

An investor buys two call options on Stock XYZ with exercise prices of $40 and $50 and sells one put option on Stock XYZ with an exercise price of $55.

The investor receives $20 for the put option and pays $20 for the two call options, so the net outlay for the options is zero. All options have the same expiration date.

The net profit is the payoff received from the call options minus the amount paid to the buyer on the put option.

A. If the investor's net profit at expiration is $3, what is the stock price on the expiration date?
B. If the investor's net profit at expiration is –$3, what is the stock price on the expiration date?

Part A: Let S be the stock price at the expiration date.

If S is less than $40, the payoff from the two call option is zero, and the payment to the buyer of the put option is at least $15. The net profit is less than –$15, so S is not less than $40.

If S is more than $55, the payoff from the two call option is at least $15 (from the call option with an exercise price of $40) plus $5 (from the call option with an exercise price of $50). The payment to the buyer of the put option is zero. The net profit is at least $20, so S is not more than $55.

If S is between $50 and $55, the payoff from the two call option is between $10 and $20: $10 to $15 on the call option with an exercise price of $40 and $0 to $5 on the call option with an exercise price of $50. The payment to the buyer of the put option is between $0 and $5. The net profit is between $5 and $20, so S is not between $50 and $55.

We infer that S is between $40 and $50. The payoff from the call option with an exercise price of $40 is S – $40. The payoff from the call option with an exercise price of $50 is zero. The payment to the buyer of the put option is $55 – S. The net profit is

$$S - \$40 - (\$55 - S) = 2S - \$95 = \$3 \Rightarrow S = (\$95 + \$3) / 2 = \$49$$

Part B: We infer that S is between $40 and $50. The payoff from the call option with an exercise price of $40 is S – $40. The payoff from the call option with an exercise price of $50 is zero. The payment to the buyer of the put option is $55 – S. The net profit is

$$S - \$40 - (\$55 - S) = 2S - \$95 = -\$3 \Rightarrow S = (\$95 - \$3) / 2 = \$46$$