NEAS VEE Regression Course – Spring 2011
Student Project
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Introduction: I used to work as a PhD candidate in vision science. My student project will explore a multiple linear regression between refractive error and various ocular components of the eye for infants. Human eye works similarly to a camera, while each of the ocular components makes its contributions to the development of refractive error. I will try to establish a multiple linear regression model that best represent the development of refractive error.

Method: The response variable is refractive error. The model started with all possible components, including axial length, corneal curvature, anterior chamber depth, lens thickness and vitreous chamber depth. I will remove the ocular components one by one starting with the largest insignificant p-value, fit a new regression model to the reduced model and observe the R values. The process will be repeated until all of the remaining ocular components had significant (95%) p-values. Minitab will be used for this project.

Data Source: Like I mentioned earlier, this set of data comes from an old research project and has been modified.

Data Description:

Variables	Description of Variables
RE	Refractive Error
AL	Axial Length
VC	Vitreous Chamber Depth
CC	Corneal Curvature
LT	Lens Thickness
AC	Anterior Chamber Depth

1. Regression Analysis of Full Model

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The regression equation is
RE = 39.8 - 1.48 AL - 1.28 VC - 0.116 CC + 0.777 LT + 0.583 AC

Predictor        Coef       StDev          T        P
Constant       39.815       3.484      11.43    0.000
AL            -1.4842      0.8208      -1.81    0.072
VC            -1.2843      0.8549      -1.50    0.134
CC           -0.11575     0.02413      -4.80    0.000
LT             0.7769      0.8191       0.95    0.344
AC             0.5829      0.8467       0.69    0.492

S = 1.250       R-Sq = 34.2%     R-Sq(adj) = 32.9%

Analysis of Variance

Source            DF          SS          MS         F        P
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```
Regression         5     210.491      42.098     26.94    0.000
Residual Error   259     404.726       1.563
Total            264     615.217

Source       DF      Seq SS
AL            1     120.685
VC            1      49.490
CC            1      38.792
LT            1       0.784
AC            1       0.741
```

Looks like at least 3 of the predicator variables are not statistically significant. The largest p-value comes from AC (Anterior Chamber Depth, p = 0.492). Therefore I will remove AC from the model and re-regress refractive error with the rest of the ocular components. Additionally, R-squared (34.2%) and adjusted R-squared (32.9%) are both relatively low. This is understandable because the contributing factors to refractive error could be extended to many more components such as optical index of various medium, anterior and posterior lens curvatures, gradient of lens index etc.

2. Regression Analysis Model with AC removed

```
The regression equation is
RE = 39.6 - 0.945 AL - 1.81 VC - 0.115 CC + 0.287 LT

Predictor        Coef       StDev          T        P
Constant       39.611       3.467      11.42    0.000
AL            -0.9452      0.2463      -3.84    0.000
VC            -1.8115      0.3797      -4.77    0.000
CC           -0.11503     0.02408      -4.78    0.000
LT             0.2865      0.4042       0.71    0.479

S = 1.249       R-Sq = 34.1%     R-Sq(adj) = 33.1%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         4     209.750      52.438     33.62    0.000
Residual Error   260     405.466       1.559
Total            264     615.217

Source       DF      Seq SS
AL            1     120.685
VC            1      49.490
CC            1      38.792
LT            1       0.784
```

The second regression shows that the R-squared slightly reduced while the adjusted R-squared improved; therefore this model is a slightly parsimonious model than the previous. Axial length (AL) and vitreous chamber depth (VC) have more significant p-values, the only insignificant variable would be lens thickness (LT, p = 0.479). Therefore the next variable to be removed would be LT.

3. Regression Analysis Model with AC and LT removed

```
The regression equation is
RE = 40.2 - 0.822 AL - 1.96 VC - 0.118 CC

Predictor        Coef       StDev          T        P
Constant       40.232       3.352      12.00    0.000
AL            -0.8222      0.1746      -4.71    0.000
VC            -1.9566      0.3196      -6.12    0.000
CC           -0.11812     0.02366      -4.99    0.000

S = 1.248       R-Sq = 34.0%     R-Sq(adj) = 33.2%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         3     208.967      69.656     44.75    0.000
Residual Error   261     406.250       1.557
Total            264     615.217

Source       DF      Seq SS
AL            1     120.685
VC            1      49.490
CC            1      38.79
```

As expected, the R-squared value is again slightly reduced, while the adjusted R-squared again slightly improves, therefore indicating this is an even more parsimonious model. All of the predictor variables have significant p value now. However, in the full model, VC (vitreous chamber depth) had an insignificant p value therefore I will remove VC for the next model to see what is going to happen.

4. Regression Analysis with AC, LT and VC removed:

```
The regression equation is
RE = 31.3 - 1.42 AL - 0.103 CC

Predictor        Coef       StDev          T        P
Constant       31.304       3.221       9.72    0.000
AL            -1.4212      0.1544      -9.21    0.000
CC           -0.10321     0.02512      -4.11    0.000

S = 1.332       R-Sq = 24.5%     R-Sq(adj) = 23.9%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         2     150.619      75.309     42.47    0.000
Residual Error   262     464.598       1.773
```

```
Total            264     615.217

Source       DF      Seq SS
AL            1     120.685
CC            1      29.934
```

This reduced model shows that both the R-squared and the adjusted R-squared are smaller, indicating that VC (vitreous chamber depth) is actually a significant contributor to refractive error development. Therefore this is not the best model to predict refractive error.

Conclusion: My conclusion is that the best model is the 3-variable multiple linear regression that had corneal curvature (CC), axial length (AL) and vitreous chamber depth (VC). Note that all three predictor variables have negative coefficients, meaning the higher their values, the lower the refractive error will be. This model (as well as all the other models) only explain at most 33.2% of the variance of refractive error. Like I said previously, a lot more contributing factors are not included in this model therefore this is by far the best model in predicting refractive errors. However, we can still be confident that the three variables we have here are indeed contributing significantly to refractive error development.

Data set:

RE	AL	VC	CC	LT	AC
4.25	14.22	8.61	60.61	3.29	2.31
4.00	14.97	8.25	60.31	3.69	3.02
3.25	14.57	8.52	61.19	3.48	2.56
3.50	14.71	8.68	61.56	3.32	2.71
3.38	14.50	8.25	61.17	3.63	2.62
5.25	14.37	8.48	59.21	3.23	2.62
5.50	14.54	8.76	59.28	3.52	2.31
3.38	13.90	8.66	61.12	3.38	2.60
4.25	14.56	9.01	58.37	3.37	2.06
6.88	13.76	8.26	60.85	3.26	2.19
5.50	14.50	8.93	59.86	3.25	2.33
7.25	14.01	8.33	60.85	3.18	2.48
4.50	15.18	9.40	57.91	3.27	2.50
5.13	14.19	8.42	60.25	3.51	2.26
5.50	14.83	8.78	61.83	3.42	2.51
4.88	14.58	8.51	61.54	3.37	2.69
7.25	14.33	8.37	61.25	3.33	2.62
6.38	13.99	8.56	61.46	3.26	2.16
5.75	14.06	8.62	60.85	3.38	2.06
5.63	14.57	8.57	60.14	3.25	2.74
5.50	13.68	8.16	64.10	3.32	2.26

6.00	14.38	8.75	60.09	3.52	2.09
4.63	14.69	9.01	59.25	3.37	2.30
2.19	14.84	8.70	61.30	3.52	2.61
4.13	14.14	8.38	63.38	3.34	2.41
4.50	14.37	8.60	61.06	3.40	2.33
6.31	14.85	8.95	58.60	3.36	2.51
3.06	14.46	8.76	60.37	3.51	1.92
2.19	14.77	9.03	61.19	3.26	2.47
4.75	14.22	8.47	62.88	3.33	2.42
2.81	14.40	8.68	61.37	3.25	2.46
3.63	14.54	8.76	62.50	3.24	2.53
2.75	13.98	8.30	64.20	3.53	2.13
3.81	14.37	8.28	63.44	2.55	2.54
3.13	14.58	8.76	61.37	3.29	2.53
5.44	13.81	8.30	64.53	3.34	2.17
4.44	13.97	8.35	64.50	3.10	2.52
6.50	14.11	8.46	62.18	3.51	2.14
5.00	14.01	8.29	63.38	3.41	2.38
6.44	14.27	8.37	61.10	3.44	2.46
3.75	14.89	9.20	57.00	3.59	2.09
6.06	13.87	8.43	60.52	3.32	2.12
5.63	13.48	7.98	64.47	3.42	2.07
3.75	14.17	8.60	63.09	3.05	2.51
2.25	14.17	8.68	64.75	3.01	2.48
2.13	14.43	8.99	63.14	2.82	2.62
1.94	15.25	9.03	62.04	3.54	2.67
-0.06	14.96	9.21	61.06	3.15	2.59
2.13	14.52	8.78	63.32	3.25	2.48
1.94	14.62	9.18	61.12	2.87	2.54
2.13	14.87	8.74	61.62	3.54	2.58
5.25	13.95	8.24	64.51	3.38	2.37
4.38	14.54	8.64	61.21	3.45	2.53
1.88	14.22	8.65	62.85	3.16	2.39
5.25	14.76	9.04	59.56	3.25	2.46
5.00	14.00	8.58	62.30	3.00	2.41
9.25	13.77	8.35	61.25	3.09	2.32
8.50	14.10	8.19	60.23	3.33	2.58
2.88	14.70	8.86	60.98	3.52	2.31
2.44	15.06	9.03	61.36	3.32	2.87
6.00	14.68	8.60	61.29	3.33	2.74
2.75	14.44	8.71	60.91	3.48	2.24

3.56	14.56	8.90	60.60	3.54	2.46
3.25	14.25	8.64	63.50	3.22	2.38
5.13	14.49	8.71	61.33	3.41	2.37
2.50	14.36	8.68	64.26	3.30	2.38
6.25	13.73	8.12	63.61	3.25	2.36
5.38	14.64	8.72	59.62	3.50	2.41
3.38	14.01	8.30	64.46	3.48	2.23
6.50	13.30	7.93	66.01	3.05	2.31
3.38	14.07	8.36	64.66	3.46	2.24
3.31	13.83	8.61	63.37	3.10	2.11
4.94	13.58	8.49	64.23	3.14	1.94
3.50	14.22	8.88	64.32	3.16	2.18
4.00	14.84	8.87	60.38	3.41	2.54
5.75	14.46	8.66	60.77	3.40	2.50
5.50	14.35	8.49	60.79	3.61	2.28
3.75	14.39	8.65	61.48	3.16	2.57
2.69	14.26	8.60	62.78	3.09	2.56
6.25	13.79	8.52	61.31	2.83	2.43
4.75	14.00	8.11	64.20	3.45	2.44
4.94	14.03	8.20	63.29	3.30	2.53
3.63	13.68	8.26	65.09	3.21	2.20
5.50	14.32	8.41	62.82	3.12	2.78
3.63	14.56	8.85	63.35	3.28	2.42
4.75	14.52	8.18	61.42	3.47	2.86
3.13	14.53	8.31	62.39	3.54	2.68
3.75	13.93	8.56	64.85	2.94	2.42
3.38	13.62	8.18	67.43	3.03	2.40
3.69	13.90	8.17	65.03	3.51	2.21
3.06	14.14	8.46	64.35	3.26	2.41
4.13	14.52	8.51	64.35	3.51	2.49
4.25	14.53	8.79	63.05	3.22	2.51
4.25	14.75	8.66	61.65	3.10	2.98
5.19	14.38	8.41	61.58	3.53	2.36
3.38	14.52	8.44	63.50	3.31	2.75
3.19	14.25	8.33	64.39	3.19	2.72
4.00	14.70	8.94	58.79	3.42	2.33
3.75	14.22	8.32	65.45	3.22	2.65
4.13	14.26	8.60	62.89	3.46	2.20
1.75	14.11	8.57	63.44	3.00	2.57
5.75	14.42	8.19	62.85	3.52	2.70
2.31	14.58	8.51	63.95	3.33	2.80

4.81	14.53	7.99	63.68	3.65	2.86
2.06	15.00	8.88	62.74	3.52	2.57
5.31	14.75	8.71	60.91	3.47	2.56
4.19	14.27	8.63	62.21	3.28	2.35
3.94	14.68	9.03	59.62	3.32	2.32
4.25	14.19	8.32	63.00	3.50	2.36
5.81	14.15	8.71	61.35	2.99	2.45
4.25	14.25	8.50	60.71	3.45	2.28
3.56	14.33	8.32	57.52	3.46	2.54
1.75	15.11	8.76	61.25	3.55	2.79
3.50	14.43	8.79	61.96	2.94	2.70
3.63	15.07	9.07	60.98	3.40	2.59
1.88	14.96	8.79	61.00	3.24	2.57
1.81	14.52	8.87	61.27	3.15	2.48
5.06	13.51	8.34	63.06	3.32	1.85
3.88	14.64	8.55	41.94	3.32	2.74
5.56	14.18	8.56	63.93	2.85	2.77
4.68	14.40	8.86	61.50	3.10	2.44
3.75	14.19	8.70	61.66	3.16	2.32
6.25	14.06	8.60	60.19	3.35	2.12
4.19	14.53	8.79	60.83	3.11	2.62
5.63	14.49	8.77	61.48	3.36	2.35
2.44	14.72	8.85	60.65	3.34	2.52
4.13	14.49	8.52	61.20	3.39	2.58
4.63	14.39	8.03	64.90	3.48	2.87
4.88	14.74	8.43	60.59	3.47	2.83
4.06	14.88	8.56	60.75	3.53	2.79
5.00	14.50	8.32	59.44	3.26	2.68
4.19	14.47	8.67	63.20	3.01	2.77
3.38	14.92	8.84	60.21	3.53	2.51
4.81	14.40	8.53	60.62	3.33	2.52
3.81	15.07	8.70	60.21	3.62	2.75
5.69	14.41	8.26	62.73	3.49	2.66
4.38	15.34	8.71	58.58	3.80	2.81
4.00	14.43	8.56	58.29	3.18	2.68
3.19	15.09	9.13	60.48	3.21	2.59
4.88	14.67	8.54	60.14	3.32	2.80
2.94	14.89	8.64	61.23	3.46	2.78
2.44	14.89	8.48	63.24	3.45	2.92
4.06	14.17	8.30	62.51	3.26	2.61
3.44	14.29	8.34	66.33	3.08	2.87

5.94	14.21	8.31	63.02	3.13	2.76
3.63	14.51	8.49	65.21	3.20	2.81
1.56	15.05	8.80	62.27	3.43	2.80
5.63	14.40	8.14	64.44	3.36	2.89
5.63	14.85	8.45	62.56	3.31	3.09
2.50	14.76	8.57	61.75	3.35	2.63
4.56	14.74	8.70	60.44	3.15	2.88
2.81	14.78	8.70	62.58	3.29	2.77
5.88	14.44	8.43	63.69	3.21	2.76
3.31	14.60	8.68	60.37	3.36	2.53
4.19	14.34	8.31	62.91	3.32	2.61
5.69	14.48	8.42	62.47	3.32	2.73
2.94	15.18	8.74	61.76	3.46	2.98
4.00	14.91	8.67	60.42	3.57	2.64
5.56	14.34	8.20	62.36	3.22	2.92
5.00	14.03	7.80	65.89	3.56	2.66
2.75	14.87	8.67	61.31	3.33	2.77
2.50	15.57	9.03	58.69	3.70	2.77
4.06	14.92	8.57	61.46	3.48	2.87
2.06	15.39	9.16	61.25	3.45	2.87
5.13	15.31	8.84	58.25	3.59	2.87
5.25	15.01	8.50	61.56	3.49	3.02
2.50	15.21	8.62	61.42	3.42	3.16
4.69	15.18	8.82	60.61	3.35	3.01
3.75	15.35	8.48	60.19	3.81	3.04
3.50	15.35	8.91	58.98	3.25	3.15
1.88	15.14	8.69	61.90	3.55	2.89
2.00	15.57	8.96	58.94	3.77	2.83
5.00	14.78	8.62	60.71	3.33	2.82
3.25	14.32	8.32	61.38	3.33	2.67
2.75	15.48	9.18	59.48	3.38	2.91
5.13	15.05	8.64	59.25	3.47	2.93
3.13	14.84	8.65	61.02	3.15	2.93
3.19	15.19	8.76	61.27	3.41	2.96
3.94	15.59	8.75	59.62	3.46	3.37
3.06	15.47	8.67	60.65	3.54	3.25
2.88	15.15	8.81	61.40	3.44	2.90
9.13	14.71	8.46	58.16	3.60	2.65
2.06	15.70	9.44	58.29	3.62	2.64
2.75	15.59	9.09	58.50	3.47	2.96
3.06	15.56	8.91	58.52	3.83	2.82

3.69	15.78	9.02	56.44	3.71	2.99
2.88	14.97	8.39	61.87	3.75	2.81
3.00	15.88	8.90	58.77	3.80	3.06
5.25	15.58	8.63	58.96	3.72	3.72
3.08	16.13	8.93	58.44	3.91	3.29
3.56	15.65	8.98	58.46	3.74	2.93
3.94	15.57	8.69	58.75	3.72	3.15
2.94	16.11	9.00	57.79	3.79	3.31
5.83	15.42	8.72	58.97	3.66	3.04
3.94	15.69	8.92	58.12	3.76	3.01
4.17	14.76	8.58	61.84	3.24	2.94
2.81	15.69	8.70	59.79	3.78	3.20
2.44	15.34	8.42	60.60	3.87	3.05
4.56	15.02	8.25	61.90	3.54	3.25
2.63	15.64	9.11	60.02	3.63	2.90
2.75	16.13	8.89	59.48	3.80	3.44
5.56	15.73	8.71	57.54	3.77	3.24
2.56	15.40	8.51	61.30	3.70	3.19
3.00	15.46	8.81	59.35	3.76	2.89
5.00	14.69	7.87	63.29	3.74	3.08
4.25	15.54	9.11	58.51	3.57	2.86
6.44	15.25	8.48	60.10	3.62	3.16
4.75	16.07	9.07	56.63	3.82	3.18
4.81	14.93	8.09	63.15	3.57	3.26
2.63	16.06	9.01	57.14	3.90	3.14
0.63	16.16	9.47	56.98	3.79	2.89
5.00	14.48	8.21	61.35	3.44	2.83
3.75	15.48	8.66	57.29	3.86	2.96
4.63	14.14	7.90	66.31	3.18	3.05
2.00	16.33	8.94	58.03	3.90	3.48
1.19	15.93	9.29	58.62	3.52	3.12
3.75	15.15	8.56	60.83	3.63	2.95
8.88	14.27	7.80	58.38	3.68	2.77
0.81	14.99	8.47	63.90	3.69	2.89
1.19	15.73	8.99	57.94	3.87	2.86
4.50	15.40	8.64	59.37	3.75	3.01
4.69	14.72	8.36	61.09	3.45	2.92
3.19	15.25	8.35	60.19	3.83	3.15
1.75	15.68	9.39	58.17	3.29	3.00
1.63	16.16	9.29	58.47	3.76	3.10
2.06	15.56	8.51	59.44	3.91	3.14

0.44	15.59	8.91	60.91	3.65	3.02
5.63	14.99	8.50	57.20	3.90	2.59
5.06	14.79	8.46	58.77	4.00	2.34
4.88	15.07	8.82	56.81	4.03	2.23
2.06	14.99	8.63	59.48	4.03	2.34
5.06	14.60	8.37	58.48	4.11	2.12
5.13	14.87	8.62	59.04	3.95	2.31
4.13	14.90	8.74	58.16	3.88	2.27
1.94	14.73	8.28	61.00	3.93	2.53
4.13	15.17	8.80	57.52	4.00	2.38
8.13	13.94	7.52	58.37	3.86	2.57
6.50	14.78	8.68	56.93	3.80	2.30
4.94	14.82	8.47	58.92	3.94	2.41
4.50	15.04	8.79	57.04	3.98	2.28
4.88	14.75	8.55	59.23	3.86	2.35
4.50	14.95	8.27	60.90	4.04	2.64
5.19	15.09	8.44	59.19	4.07	2.58
1.75	15.01	8.55	61.56	3.88	2.57
2.75	15.27	8.79	59.75	3.70	2.78
3.31	14.89	8.56	60.12	3.75	2.58
3.81	14.73	8.48	103.01	3.65	2.60
3.75	15.45	8.95	58.41	3.72	2.77
4.94	15.15	8.59	57.71	4.05	2.51
2.63	15.12	8.49	60.46	3.99	2.65
3.19	15.48	8.82	58.60	3.80	2.86
6.38	14.48	8.02	60.15	3.96	2.50
3.38	14.78	8.52	60.88	3.68	2.57
4.31	15.32	8.64	58.27	3.83	2.85
4.13	15.13	8.80	59.06	3.76	2.57
5.56	14.60	8.08	61.33	3.94	2.57
6.81	14.29	8.36	60.11	3.73	2.19
1.69	14.94	8.45	61.23	3.99	2.51
2.19	15.74	8.91	58.67	4.01	2.81
3.88	15.39	8.58	58.10	4.01	2.79
3.88	15.55	8.92	55.81	3.96	2.68
3.38	15.30	8.48	58.75	4.07	2.75
2.06	14.98	8.66	63.48	3.75	2.57
6.25	15.05	8.50	58.54	3.67	2.88
2.88	15.04	8.75	60.27	3.94	2.35