# Time Series Project 

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## Estimation of Jet Fuel Prices

## Introduction

Historically, the prices of jet fuel are showing much volatility especially over the past few years the variability of fuel prices causes serious problem for private airlines as well as military budget. If the future prices are predicted through regression analysis and time series modeling it would not only assist in lessening the stress caused due to unknown future fuel prices, but also help investors to hedge themselves against variable future prices
The project aims to model the price of jet fuel per gallon. Initially, the data is thoroughly examined and several models including $\operatorname{AR}(1), \operatorname{AR}(2), \operatorname{AR}(3)$ were analyzed to estimate the best fitted model.

## Data

The data of monthly jet fuel prices from January 1995 to December 2005 (132 data points) is used. The data is taken from
http://www.indexmundi.com/commodities/?commodity=jet-fuel\&months=360. The graph of the raw data is shown below:


Please see the tab "data (Raw Data)" for the method to produce the graph.

The data is checked fro stationarity before attempting to fit an ARIMA model. Hence, sample autocorrelation is performed to determine the stationarity of the series.


Please refer to the "data" tab in Excel. As we can see from the graph, the correlation does not reach 0 until around lag 57 (must reach 0 "quickly" which it clearly does not in this case). Then it goes below 0 and levels out to zero at lag 130. I will look at first differences to see if it is stationary.

## $1^{\text {st }}$ Difference



And its corresponding correlograms is shown as below:


The 1st difference appears to be stationary because it does not depict any fluctuations with respect to time. The autocorrelation both go to zero quickly and oscillate around zero and decrease oscillation as lag increases.

Several models such as $\operatorname{AR}(1), \operatorname{AR}(2)$ and $\operatorname{AR}(3)$ models will be fit to the $1^{\text {st }}$ difference

## Parameterization of the Model

After having stationary data, we will fit our data using an autoregressive model (AR (p)), using $p=1,2$ and 3 . Since we are using the first difference, this is equivalent to ARIMA ( $\mathrm{p}, 1,0$ ) models.
By Using Excel's regression data analysis add-in, the following is a summary of the regression results along with the resulting AR equations:

The detail results are attached in Excel sheets, (see tabs $1^{\text {st }}$ Regression, $2^{\text {nd }}$ Regression and 3rd Regression)

## AR (1)

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.017676993 |
| R Square | 0.000312476 |
| Adjusted R Square | -0.00743704 |
| Standard Error | 5.784822444 |


|  | Coefficients | Standard <br> Error | $\boldsymbol{t}$ Stat | P-value | Lower <br> $\mathbf{9 5 \%}$ | Upper <br> $\mathbf{9 5 \%}$ | Lower <br> $\mathbf{9 5 . 0 \%}$ | Upper <br> $\mathbf{9 5 . 0 \%}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 0.6656 | 0.5088 | 1.3081 | 0.1932 | $(0.3411)$ | 1.6722 | $0.3411)$ | 1.6722 |
| $\mathbf{Y}_{(\mathrm{T}-1)}$ | 0.0177 | 0.0880 | 0.2008 | 0.8412 | $(0.1565)$ | 0.1919 | $(0.1565)$ | 0.1919 |

So the first equation is:
$\boldsymbol{A R}$ (1): $\quad \mathrm{Y}_{\mathrm{t}}=0.01768 \mathrm{Y}_{\mathrm{t}-1}+0.66558+\mathrm{e}_{\mathrm{t}}$


The adjusted $\mathrm{R}^{2}$ is negative showing the series shows negligible or no independence on each other.

## AR (2):

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.243208115 |
| R Square | 0.059150187 |
| Adjusted R Square | 0.044333655 |
| Standard Error | 5.655580522 |
| Observations | 130 |


|  | Coefficients | Standard <br> Error | t Stat | P-value | Lower <br> 95\% | Upper <br> $\mathbf{9 5 \%}$ | Lower <br> $\mathbf{9 5 . 0 \%}$ | Upper <br> $\mathbf{9 5 . 0 \%}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 0.9811 | 0.5113 | 1.9190 | 0.0572 | $(0.0306)$ | 1.9928 | $(0.0306)$ | 1.9928 |
| X Variable 1 | 0.0268 | 0.0861 | 0.3113 | 0.7561 | $(0.1436)$ | 0.1973 | $(0.1436)$ | 0.1973 |
| X Variable 2 | $(0.3185)$ | 0.1130 | $(2.8183)$ | 0.0056 | $(0.5421)$ | $(0.0949)$ | $(0.5421)$ | $(0.0949)$ |

The second equation is:
$\boldsymbol{A R}(2): \mathrm{Y}_{\mathrm{t}}=0.0268 \mathrm{Y}_{\mathrm{t}-1}-0.3185 \mathrm{Y}_{\mathrm{t}-2}+0.9811+\mathrm{e}_{\mathrm{t}}$


AR(3)

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.259073 |
| R Square | 0.067119 |
| Adjusted R Square | 0.044729 |
| Standard Error | 5.675429 |
| Observations | 129 |


|  | Coefficients | Standard <br> Error | t Stat | P-value | Lower <br> $\mathbf{9 5 \%}$ | Upper <br> $\mathbf{9 5 \%}$ | Lower <br> $\mathbf{9 5 . 0 \%}$ | Upper <br> $\mathbf{9 5 . 0 \%}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 1.0987 | 0.5254 | 2.0912 | 0.0385 | 0.0589 | 2.1384 | 0.0589 | 2.1384 |
| X Variable 1 | 0.0026 | 0.0896 | 0.0291 | 0.9768 | $(0.1747)$ | 0.1799 | $(0.1747)$ | 0.1799 |
| X Variable 2 | $(0.2933)$ | 0.1162 | $(2.5248)$ | 0.0128 | $(0.5232)$ | $(0.0634)$ | $(0.5232)$ | $(0.0634)$ |
| X Variable 3 | $(0.1243)$ | 0.1217 | $(1.0217)$ | 0.3089 | $(0.3651)$ | 0.1165 | $(0.3651)$ | 0.1165 |

The third equation is:
$\boldsymbol{A R}(3): \mathrm{Y}_{\mathrm{t}}=0.0026 \mathrm{Y}_{\mathrm{t}-1}-0.2933 \mathrm{Y}_{\mathrm{t}-2}-0.1243 \mathrm{Y}_{\mathrm{t}-3}+1.0987+\mathrm{e}_{\mathrm{t}}$


## Results

| Table | Sum of <br> Coefficients | R- <br> Squared | Adjusted R- <br> Squared | Durbin-Watson <br> Statistic | Box Pierce | Chi-Square <br> $\mathbf{1 0 \%}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{AR}(1)$ | 0.6833 | 0.000313 | -0.00743 | 1.99227 | 51.88 | 147.80 |
| $\operatorname{AR}(2)$ | 0.6895 | 0.05915 | 0.044333 | 2.04248 | 41.5595 | 147.8048 |
| $\operatorname{AR}(3)$ | 0.6837 | 0.06712 | 0.044729 | 2.01181 | 41.08312 | 147.8048 |

The above table shows that the sum of coefficients for each model is less than 1 which shows that the models are stationary. Moreover, the Durbin-Watson statistic is around 2 for each, suggesting no serial correlation. It is also observed thatthe Box-Pierce Q statistics are lower than Chi-Squared (10\%) critical value. The null hypothesis therefore, cannot be rejected which states that the residual are formed by a white noise process.

## Selection of Model:

Based on R-squared and Adjusted R-squared, $\operatorname{AR}(2)$ and $\operatorname{AR}(3)$ is much better than $\operatorname{AR}(1)$. Moreover, the value of box pierce test \& adjusted $R^{2}$ is almost same for $\operatorname{AR}(2)$ and AR(3). Based on the Principle Of Parsimony which states given a choice of two almost equivalent model selection, select the simpler one, $\boldsymbol{I}$ will select $\boldsymbol{A R}(2)$
$\boldsymbol{A R}(2): \mathrm{Y}_{\mathrm{t}}=0.0268 \mathrm{Y}_{\mathrm{t}-1}-0.3185 \mathrm{Y}_{\mathrm{t}-2}+0.9811+\mathrm{e}_{\mathrm{t}}$

