TSS module 19 Seasonal models practice problems

(The attached PDF file has better formatting.)

Background: Rationale for single period and seasonal ARIMA parameters

Several common scenarios for seasonal model are listed below; many others occur in practice

1. *Weather:* Seasonality is a yearly phenomenon: August temperature may be 40° higher than average, and February temperature may be 40° lower than average. Cryer and Chan use sine and cosine processes to model daily temperature; other statisticians de-seasonalize the daily temperature. A seasonal ARIMA parameter is not appropriate: a hotter than usual August last year doesn't imply a hotter August this year.

2. *Seasonal sales:* Children's toys in December; Turkeys in November, travel tickets in August; ski gear in winter months; and other seasonal items use last year's sale to forecast sales this year. Use seasonal ARIMA parameters, such as a 12 month autoregressive parameter Φ, or de-seasonalize the data.

3. *Annual contracts:* rent, insurance, leases: insurance is for annual term; premium was high in Month Z last year, we expect it to be high this year, since most policies are renewed. Use seasonal ARIMA parameters (autoregressive parameters), not de-seasonalized data.

For your student project, examine whether the time series is seasonal from the autocorrelation function (the correlogram). Explain the type of seasonality, and say how you deal with it.

Exercise 19.1: Seasonal moving average process

- A seasonal moving average process has $\theta = \theta_1 = -0.6$ and $\Theta = \Theta_1 = -0.8$.
- This is a monthly model, with seasonality of 12 months.

A. What is ρ_1?
B. What is ρ_{11}?
C. What is ρ_{12}?
D. What is ρ_{13}?

[Cryer and Chan drop the subscript if there is only one θ or Θ parameter. They use *s* for the number of periods in the season. The notation is confusing at first, but the logic is straight-forward.]

Part A: The autocorrelation of lag 1 is not affected by the seasonal parameter: $\rho_1 = -\theta / (1 + \theta^2) = 0.441$.

See Cryer and Chan, chapter 10, bottom of page 230, equation 10.2.3.

Part B: The autocorrelation of lag 11 is the product of the non-seasonal and seasonal effects:

$$\rho_{11} = \rho_{13} = \theta\ \Theta \ /\ [\ (1 + \theta^2)\ (1 + \Theta^2)\ ] = 0.215.$$

See Cryer and Chan, chapter 10, bottom of page 230, equations 10.2.4.

Part C: The autocorrelation of lag 12 reflects the seasonal parameter only: $\rho_{12} = \Phi\ \ -\Theta / (1 + \Theta^2)$ = 0.488.

See Cryer and Chan, chapter 10, top of page 231, equation 10.2.5.

Part D: The autocorrelation of lag 13 equals the autocorrelation of lag 11.

Intuition: In a stationary process, the autocorrelation of lag k = the autocorrelation of lag –k, so the seasonal autocorrelation plus the lag for the regular autocorrelation is like the seasonal autocorrelation minus the lag for the regular autocorrelation.

Final exam problems give other values for θ and Θ. The model is multiplicative; see Part B above.

Exercise 19.2: Seasonal mixed moving average / autoregressive process

- A seasonal moving average process has $\theta = \theta_1 = -0.6$ and $\Phi = \Phi_1 = +0.8$.
- This is a monthly model, with seasonality of 12 months.

A. What is ρ_1?
B. What is ρ_{11}?
C. What is ρ_{12}?
D. What is ρ_{13}?

[Cryer and Chan drop the subscript if there is only one θ or Φ parameter. They use *s* for the number of periods in the season. The notation is confusing at first, but the logic is straight-forward.]

Part A: The autocorrelation of lag 1 is not affected by the seasonal parameter: $\rho_1 = -\theta / (1 + \theta^2) = 0.441$.

See Cryer and Chan, chapter 10, bottom of page 230, equation 10.2.3.

Part B: The autocorrelation of lag 11 is the product of the non-seasonal and seasonal effects:

$$\rho_{11} = \rho_{13} = -\theta\,\Phi /(1 + \theta^2) = 0.353.$$

See Cryer and Chan, chapter 10, page 232, equation 10.2.11. Cryer and Chan give the formula for ρ_{12k-1} and ρ_{12k+1}; this exercise has $k = 1$.

Part C: The autocorrelation of lag 12 reflects the seasonal parameter only: $\rho_{12} = \Phi = 0.800$.

See Cryer and Chan, chapter 10, top of page 232, equation 10.2.11.

Part D: The autocorrelation of lag 13 equals the autocorrelation of lag 11.

Final exam problems give other values for θ and Φ. The model is multiplicative; see Part B above.