Corporate finance, Module 7, CAPM, covariance, practice problems

(The attached PDF file has better formatting.)

** Exercise 7.1: Beta

The risk-free rate equals the market risk premium, and the *covariance* of a stock's returns with the overall market return equals the *variance* of the overall market returns. Because of changes in operations, the covariance of the stock's returns with the overall market returns decreases 10% but the variance of the market return does not change.

A. What is the stock's beta before the change in operations?
B. What is the stock's expected return as a function of the risk-free rate before the change in operations?
C. What is the stock's beta after the change in operations?
D. What is the stock's expected return as a function of the risk-free rate after the change in operations?
E. What is the percentage change in the stock's expected return from the change in opearations?

Part A: The CAPM beta is the covariance of the stock's return with the market return divided by the variance of the market return. In this exercise, the covariance of a stock's returns with the overall market return equals the variance of the overall market returns, so the CAPM beta is one.

Part B: If the risk-free rate is R, the market risk premium is also R. Before the change in operations, the stock's expected return is R + 1.00 × R = 2R.

Part C: If the covariance of the stock's return with the market return decreases 10% and the variance of the market return does not change, the CAPM beta decreases 10%.

Part D: After the change in operations, the stock's expected return is R + 0.90 × R = 1.90R.

Part E: The percentage change in the expected return is 1.90 / 2.00 – 1 = –5.00%.

** Exercise 7.2: Beta

The standard deviation of overall market returns is 41%. The correlation of an oil company's stock's returns with the overall market return is 59%, and the standard deviation of the oil stock's returns is 71.5%.

Because of war in Iran, the standard deviation of the oil company's stock's return increases 25%. The variance of the market return and the correlation of the stock's return with the market return do not change.

A. What is the formula for the CAPM beta in terms of standard deviations and correlations?
B. If the standard deviation of the stock's return increases P%, what is the change in its CAPM beta?

Part A: The CAPM beta is the covariance of the stock's return with the market return divided by the variance of the market return. The correlation is the covariance divided by the product of the standard deviations of the two variables. The standard deviation is the square root of the variance.

The CAPM beta (β) = covariance(r_s, r_m) / $\sigma^2(r_m)$ = [$\rho(r_s, r_m) \times \sigma(r_s) \times \sigma(r_m)$] / $\sigma^2(r_m)$ = [$\rho(r_s, r_m) \times \sigma(r_s)$] / $\sigma(r_m)$

Part B: If all else remains the same, the CAPM β is proportional to the standard deviation of the stock's return. If the standard deviation increases 25%, the β increases 25%. The figures in the exercise (41%, 59%, 71.5%) do not affect the solution.

** Exercise 7.3: Betas

The CAPM beta of stock W is 1.200, the CAPM beta of stock Y is 0.800, and the risk-free rate equals the market risk premium.

A. What is the relation of the covariance of Stock W's return with the market return to the covariance of Stock Y's return with the market return?
B. What is the relation of the expected return on Stock W to the expected return on Stock Y?
C. What is the relation of the excess of the expected return for stock W over the risk-free rate ($r_w - r_f$) to the excess of the expected return for stock Y over the risk-free rate ($r_y - r_f$)?

Part A: The CAPM beta is the covariance of returns for the stock with the overall market returns divided by the variance of the overall market returns. The beta of stock W is 50% higher than the beta of stock Y, so the covariance is 50% higher.

Part B: Let the risk-free rate be R, so the market risk premium is also R.

● The expected return on Stock W is R + 1.2 × R = 2.2 × R.
● The expected return on Stock Y is R + 0.8 × R = 1.8 × R.

The ratio of the expected returns is 2.2R / 1.8R = 1.222

Part C: The risk-free rate reflects inflation, the time value of money, and the effect of taxes on investment income. The CAPM beta measures the additional return required for risky common stock over risk-free bonds.

From the CAPM equation:

● $r_Y - r_f = \beta_Y \times$ market risk premium
● $r_W - r_f = \beta_W \times$ market risk premium

$\Rightarrow (r_W - r_f) / (r_Y - r_f) = \beta_W / \beta_y = 1.200 / 0.800 = 1.5 = 50\%$ greater.

** Exercise 7.4: CAPM Beta and correlation

The variance of the overall market return is 25%, the variance of returns on stock ABC is 49%, and the stock's beta is 0.840.

A. What is the standard deviation of the overall market return?
B. What is the standard deviation of the return on stock ABC?
C. What is the correlation of the returns on stock ABC and the overall market return?

Part A: The standard deviation of the overall market return is the square root of the variance:

$$\sigma(r_m) = \sqrt{25\%} = 50\%$$

Part B: The standard deviation of the return on stock ABC is $\sigma(r_s) = \sqrt{49\%} = 70\%$.

Part C: The CAPM beta is the covariance of the stock's return with the market return divided by the variance of the market return. The correlation is the covariance divided by the product of the standard deviations of the two variables. The standard deviation is the square root of the variance.

The CAPM beta (β) = covariance$(r_s, r_m) / \sigma^2(r_m)$ = $[\rho(r_s, r_m) \times \sigma(r_s) \times \sigma(r_m)] / \sigma^2(r_m)$ = $[\rho(r_s, r_m) \times \sigma(r_s)] / \sigma(r_m)$

$$0.840 = \rho(r_s, r_m) \times 0.70 / 0.50 \Rrightarrow$$
$$\rho(r_s, r_m) = 0{,}.840 \times 0.50 / 0.70 = 60.00\%$$